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JOURNAL OF EDUCATIONAL RESEARCH

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JOURNAL *of* EDUCATIONAL RESEARCH

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ARITHMETICAL CONCEPTS OF CHILDREN*

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I. INTRODUCTION

UNDER many present-day methods of teaching arithmetic, the child is forced to acquire discrete *facts* which mean that learning becomes chiefly a process of memorizing instead of discovering solutions. Learning is forced out of relation to the normal growth processes. Many classroom teachers have discovered for themselves that drill methods in arithmetic can lead to unfortunate consequences, but the law of repetition in educational psychology was seldom challenged either in practice or in theory, until recently. It is the contention of the organismic psychologist that this law is not well founded and that the child's understanding of problem-tasks is more important than mere repetition.

If we are to promote child mastery of arithmetic, then we must determine first of all what number-ideas children have and in what situations children employ mathematical concepts. Wheeler and Perkins assert that "Learning arithmetic begins with the child's undifferentiated concept of number, size, magnitude, and quantity. Originally his quantitative world contains no idea of specific sizes or quantitative value. His first differentiation is into more and less, a gross differentiation of a quantity of field or range."[1] The child supposedly makes these quantitative judgments of *more* or *less* very early for he makes use of them when he first handles objects. It was the purpose of our study to test kindergarten, first grade, and second grade children for their in-

*The investigation reported in this paper was made with the coöperation of F. T. Perkins and R. H. Wheeler of the University of Kansas, and also with the assistance of Ursula Henley and Helen Gaw of the Lawrence, Kansas, Public Schools. The writer is indebted especially to F. T. Perkins for suggestions and criticisms in the preparation of the manuscript.

[1]Wheeler, R. H., and Perkins, F. T. *Principles of Mental Development* (New York: Thomas Y. Crowell, 1932), p. 480.

sight into quantitative relations of *more* and *less*. If these judgments are the first number responses, they would be very accurately developed by the time the child reaches the ages of four or five years.

Another related problem concerns the mathematical vocabulary of children four to eight years of age. These children would not only understand words expressing the quantitative relations *more* and *less*, but ought to use them in making their wants known to their playmates and parents. It is to be expected that words having the meanings of *equal, same, add,* and *subtract* would be more difficult and therefore not understood so well nor used so frequently as words denoting *more* and *less*.

Wheeler and Perkins maintain that number ideas must first be well differentiated before the teacher should give the child addition problems in the form of addition 'facts.' It would appear, also, that the attempt to have the child count is a mistaken procedure until the child gains an adequate conception of the magnitude or quantity which he is differentiating. True counting is not the cause of number development, but is the result of the development. As the cardinal idea of number differentiates for the child, there occurs, simultaneously, a realization of the various relationships of number as a part of a more inclusive concept, the number series.

It must be established how large a quantity children can deal with and also, if possible, in what way they actually work with quantities. We aimed, then, to discover in children four to eight years of age: 1. The responses to quantitative situations of *more* and *less*. 2. The understanding and use of words denoting *more, less, many, equal,* and *same*. 3. The limits, perceptually, beyond which they cannot make distinctions between quantities. 4. The manner in which they go about the task of making distinctions between quantities.

II. EXPERIMENTAL METHOD

A preliminary study was made to determine the best method of presenting quantities to the child so that he might judge which was larger or largest. It was decided that the child should be presented two quantities in the form of two groups of blocks.

The questions asked the children in the first study were: Are the two piles of blocks equal? The same? Can you make them equal? Which pile has the most blocks? The least number of blocks? Which pile has more blocks? Less blocks? It was anticipated that children from the ages of four to eight years of age would not distinguish sharply the comparative degree (more) from the superlative degree (most). It was found

that many children would use *more* and *most* as well as *less* and *least* interchangeably. Questions involving the words *most* and *least* were understood in many cases when the words *more* and *less* were not understood.

The main study was divided into two experiments, A and B. Three tests with similar materials were used in both A and B. In Test I the experimenter presented two irregular groups of blocks before the individual subject. These blocks were unpainted and were one inch square in size. The following are the number of blocks that were used in the groups of the various combinations: 3-6; 5-7; 2-5; 6-8; 1-4; 7-10; 3-3; 5-8; 2-6; 8-8; 5-6; 8-10; 1-2; 9-10; 6-6; 7-9; 3-4; 6-7; 1-5; 10-10; 4-4; 8-9; 1-6; 3-5; 4-7; 2-4; 9-9; 4-6; 5-5; 7-7; 2-3; 1-1; 4-5; 7-8; 2-2; and 1-3. The combinations were presented in the order given above. If the child hesitated, if he counted, or if he insisted upon manipulating the blocks in some manner before making a response, it was recorded on the individual subject's data sheet. The children in Experiment A were asked the question, "Which pile has the most blocks?" If the child had difficulty with this question, the following were given: "Are the piles of blocks the same?" "Does one pile have more blocks than the other?" "How many blocks does this pile have?"

It was desired to check the results obtained on Test I by varying the materials (blocks). In Test II (see Table I) each pile had both small and large blocks in it. The small blocks were unpainted and were one inch square while the large blocks were unpainted and were two inches square. In Test III (see Table I) painted blocks one inch square were employed. White, blue, green, yellow, and red colors were used. The experimenter asked the child exactly the same types of questions on Tests II and III as on Test I. One subject took the first half of Test II and the second half of Test III, while the next subject took the second half of Test II and the first half of Test III. The results from Tests II and III were combined.

A number of precautions were taken to prevent extraneous factors from affecting the judgments. First, the larger group was placed to the right or to the left of the subject in random order. Second, the smaller group was placed first in some instances and the larger one first in other instances. Third, the experimenter quickly learned the combinations well enough so that it was possible to keep a move ahead of the subject; changes were performed more rapidly than the latter could possibly count. In the majority of cases, it may be added, the children were intent on the problem and not on ways of getting around the problem. It is not likely, under these conditions, that cues gave

away the correct answer. Whenever a subject failed to coöperate, he merely resorted to guessing and there was no instance when guessing gave consistently correct results.

TABLE I

Test II					Test III									
Small (S) and Large (L) Block Combinations					Colors in Combinations—White(W), Blue (B), Green (G), Yellow (Y), Red (R)									
Combination	S	L	S	L	Combination	W	B	G	Y	R	W	B	G	Y	R
5- 7	3	2	5	2	5- 7	1	1	1	1	1	2	1	1	1	2
2- 5	1	1	2	3	2- 5	0	1	1	0	0	2	0	1	1	1
7-10	4	3	6	4	7-10	2	1	2	1	1	3	4	1	1	1
3- 3	2	1	3	0	3- 3	0	0	1	2	0	1	1	0	0	1
5- 8	4	1	5	3	5- 8	2	0	2	0	1	0	4	1	3	0
8- 8	6	2	1	7	8- 8	1	3	0	3	1	0	0	5	0	3
5- 6	3	2	2	4	5- 6	1	1	1	1	1	2	1	1	2	0
8-10	4	4	5	5	8-10	1	1	1	1	4	4	2	1	2	1
6- 6	5	1	3	3	6- 6	1	1	4	0	0	2	0	0	1	3
9-10	6	3	4	6	9-10	1	3	2	1	2	1	2	3	1	3
7- 9	5	2	6	3	7- 9	3	1	1	1	1	2	0	3	2	0
3- 4	2	1	1	3	3- 4	0	0	2	0	1	1	1	1	1	0
8- 9	7	1	3	6	8- 9	2	3	1	1	1	1	1	0	4	3
4- 4	2	2	3	1	4- 4	1	0	2	1	0	0	1	0	0	3
2- 4	1	1	1	3	2- 4	1	0	0	1	0	0	2	1	0	1
4- 7	2	2	4	3	4- 7	1	3	0	0	0	0	1	3	2	1
5- 5	4	1	3	2	5- 5	2	1	1	1	0	1	1	1	1	1
4- 6	1	3	4	2	4- 6	1	1	1	1	0	2	0	1	0	3
7- 8	6	1	5	3	7- 8	2	1	0	0	4	0	3	1	3	1
2- 3	1	1	2	1	2- 3	0	1	0	1	0	1	1	0	0	1
6- 7	5	1	4	3	6- 7	1	2	1	1	1	2	1	2	1	1
4- 5	1	3	4	1	4- 5	1	1	1	1	0	1	1	1	1	1
7- 7	6	1	5	2	7- 7	2	0	1	2	2	1	1	1	2	2

In Experiment B, the same materials as in A were employed with the following exceptions. It was found necessary to change the proportion of big to little blocks in the combinations of Test II. In too many of these combinations it was found that the greater number of big blocks appeared in the larger group. Since the results indicated that the big blocks were influencing judgments when the child was to look for the larger group, the combinations were changed in such a fashion that a guess on this basis alone would be correct only 50 percent of the time. Half of the group were given Test II and the other half were given Test III. The questions asked the child regarding the combinations constitute the greatest difference in procedure between Experiments A and B. A Survey of the results from Experiment A revealed that the pupils were being influenced by the type of questions asked them. When they were asked to find the group of blocks which was the larger, they were intent upon finding a difference between the groups. On the more difficult combinations, consequently, many errors were committed. In

order to check this explanation of the errors, the instructions to the subject in Experiment B were changed so that he was to look for sameness in the two groups of every combination. The questions were worded usually in one or the other of the following two forms, "Are the piles

TABLE II

EXPERIMENT A

	Pupils	Boys	Girls	Av. Age (Yrs. and Mos.)
Kindergarten	13	8	5	5- 8
First Grade	12	7	5	6-10
Second Grade	4	3	1	7-10
Total Group	29	18	11	6- 9

TABLE III

INTELLIGENCE TEST RESULTS ON THE SUBJECTS OF EXPERIMENTS A AND B

Pupil* (Experiment A)	I.Q.**	Grade in School 1932–1933	Pupil*** (Experiment B)	I.Q.	Grade in School 1932–1933
1.	...	1st	1.	96	K
2.	...	1st	2.	131	1st
3.	104	K	3.	106	K
4.	106	1st	4.	102	2nd
5.	103	K	5.	113	K
6.	105	K	6.	124	K
7.	116	2nd	7.	122	2nd
8.	105	1st	8.	100	1st
9.	100	K	9.	118	K
10.	105	1st	10.	96	1st
11.	114	K	11.	...	1st
12.	...	1st	12.	109	K
13.	117	K	13.	113	1st
14.	97	K	14.	124	K
15.	120	K	15.	91	1st
16.	109	1st	16.	101	2nd
17.	109	1st	17.	107	1st
18.	103	1st	18.	121	1st
19.	110	K	19.	109	1st
20.	114	2nd	20.	110	2nd
21.	104	1st	21.	100	1st
22.	108	2nd	22.	...	K
23.	...	...	23.	...	2nd
24.	101	K	24.	...	K
25.	101	K	25.	...	K
26.	98	1st			
27.	96	2nd			
28.	105	1st			
29.	105	K			

*The numbers in this column refer to the case numbers employed for the pupils in Experiment A.

**The I.Q. scores were obtained according to the Stanford Revision of the Binet-Simon Intelligence Test. The tests were given during the months of November and December, 1933, and also January, 1934. 89 percent of the original group in Experiment A are included; the average I.Q. was 106.1. 80 percent of the original group of Experiment B are included; the average I.Q. was 109.7.

***The numbers in this column refer to the case numbers employed for the pupils in Experiment B.

of blocks the same? Are the piles of blocks equal in number?'' It was found, as might easily be surmised, that the children generally used *same* for *equal*. There were very few instances throughout the whole investigation in which a child spontaneously used the word *equal*. The phrase, ''They're both the same,'' on the other hand, was used by the children many times throughout both Experiments A and B.

III. SUBJECTS

The subjects in Experiments A and B were pupils in the kindergarten, first grade, and second grade respectively, during the school year 1932-1933 at Lawrence, Kansas. The data regarding these subjects including intelligence test scores on the Stanford Revision, appear in Tables II to IV inclusive.

TABLE IV
EXPERIMENT B

	Pupils	Boys	Girls	Av. Age (Yrs. and Mos.)
Kindergarten	10	4	6	6-3
First Grade	10	5	5	7-0
Second Grade	5	4	1	8-0
Total Group	25	13	12	7-1

IV. QUANTITATIVE RESULTS

In Experiment A, the children were asked to look for differences in the groups of blocks in each combination while in Experiment B they were required to find sameness. Tables V and VI show the results obtained on Test I in Experiments A and B, respectively.

The first column in each table marked 'Combination' contains the equal combinations, 10-10, 9-9, 8-8, and so on, while the third column marked 'Combination' contains the unequal combinations, 9-10, 8-10, 8-9, and so on. The second and fourth columns give the percentages of error on the combinations listed in columns one and three, respectively. Each column is divided into two parts so that the average percent of error for the higher as opposed to the lower numbered combinations can be compared immediately.

The nature of the instructions, whether the experimenter has the child looking for differences or sameness, seems to play an important part as far as the child's arithmetic performance is concerned. There is a high percentage of error on the higher, even combinations in Table V. The large drop in error occurs between 5-5 and 3-3. 5-5 is perceived as unequal in 52 percent of the cases while the error on 3-3 is 14 percent. It is significant that several subjects persisted in finding dif-

TABLE V

RESULTS OF EXPERIMENT A

TEST I

Identical Combinations	Percent Error	Combination	Percent Error
10–10	86	9–10	31
9–9	72	8–10	10
8–8	86	8–9	24
7–7	66	7–10	3
6–6	66	7–9	14
		7–8	28
	Av. 75.2	6–8	3
			Av. 16.1
5–5	52	6–7	14
4–4	28	5–6	10
3–3	14	4–5	7
2–2	14	3–5	3
1–1	10	3–4	3
		2–4	3
	Av. 23.6	1–5	3
			Av. 6.1

TABLE VI

RESULTS OF EXPERIMENT B

TEST I

Identical Combinations	Percent Error	Combination	Percent Error
10–10	48	9–10	52
9–9	52	8–10	28
8–8	48	8–9	48
7–7	48	7–10	12
6–6	48	7–9	12
		7–8	28
	Av. 48.8		Av. 30.0
5–5	24	6–8	4
4–4	16	6–7	40
3–3	4	5–7	18
2–2	0	5–6	32
1–1	0	4–5	13
		3–4	4
	Av. 8.8		Av. 18.5

ferences on 3-3, 2-2, and 1-1. When looking for differences on the unequal combinations, children apparently can distinguish 6 from 7 with fair accuracy; a difference of one on 4-5 still offers some difficulty. Differences of two between the groups of blocks are perceived by 90 percent of the children. In Table VI there is a lower percentage of error on the equal combinations than in Table V. A rapid decrease in errors occurs between 6-6 and 5-5 and there is practically no change in the number of errors from 10-10 to 5-5 which is in wide contrast to the results on the equal combinations in Table V. Although the pupils in Experiment B, were slightly older on the average than those in Experiment A, this difference in age will hardly explain the variations of results on the two tests. On the unequal combinations there is on the average over twice as much error in Experiment B as on the corresponding combinations in Experiment A. The pupils are very certain of themselves only on 4-5 and 3-4.

Tables VII and VIII which give the combined results of Tests II and III in Experiments A and B, respectively, show the same general trend

TABLE VII

RESULTS OF EXPERIMENT A

TESTS II AND III COMBINED

Identical Combinations	Percent Error	Combination	Percent Error
8–8	86	9–10	11
7–7	79	8–10	11
6–6	68	8–9	22
		7–10	11
	Av. 77.5	7–9	11
		7–8	22
		6–7	11
			Av. 14.1
5–5	54	5–8	7
4–4	39	5–7	4
3–3	14	5–6	14
		4–7	4
	Av. 35.5	4–6	11
		4–5	14
		3–4	4
			Av. 8.3

TABLE VIII

RESULTS OF EXPERIMENT B

TESTS II AND III COMBINED

Identical Combinations	Percent Error	Combination	Percent Error
8–8	50	9–10	46
7–7	54	8–10	8
6–6	25	8–9	42
		7–10	13
	Av. 43.0	7–9	17
			Av. 25.2
5–5	46	7–8	42
4–4	21	6–7	29
3–3	8	5–6	17
		4–6	4
	Av. 25.0	4–5	4
		3–4	4
			Av. 16.7

of results that was obtained with Test I in Experiments A and B. There are apparently more errors made on the equal combinations and less errors on the unequal combinations on Tests II and III than was found on the equal and unequal combinations on Test I. More time was required to obtain a solution of these problems in nearly every case.

V. QUALITATIVE RESULTS

Vocabulary of the seven-year-old.—The pupil who is about to enter the second grade probably will have a general knowledge of such terms as *many, most, more*, and some understanding of *less* and *least*. Mathematical terms denoting *same* or *equal* are not likely to be comprehended: at least, the teacher cannot proceed on the assumption that the child will be able to use these terms. There was considerable difficulty in getting the pupils in this investigation to comprehend the instructions which

demanded a knowledge of *equal* or *same*. 3A (subject 3, Experiment A) failed altogether to get the idea of the questions, "Are the two piles equal?", Are they the same?". He pointed to the group which he thought was larger in every instance. 6A also had difficulty with these questions. Most pupils who did understand the term *same* would exclaim of their own accord, "They're both the same," or perhaps only the word, "None," when asked which group had the greater number of blocks.

Influence of size, color, and form of the number response.—There is little doubt but that Tests II and III offered much more difficulty to the children than did Test I. The quantitative aspect of the results revealed to some extent that such was the case, although these results shed little light on the nature of the difficulty involved. The younger children especially had difficulty with Tests II and III and would exclaim, for example, "This is hard." Each group of blocks was differentiated first by these younger children into subgroups made on the basis of size or color and this naturally made a quantitative comparison of the two groups more difficult, since quantity as such varied independently of the size and color of particular objects. As a result, the pupils who did not surmount this difficulty judged the larger group of blocks on the basis of the subgroup which appeared largest. Thus, a new type of judgment appeared. The largest subgroup of these complex patterns became the 'sign' for the largest whole; subgroups containing many large blocks, of course, carried much more weight than subgroups made up primarily of small blocks. 14A missed the combination 4-6 on Test II being influenced by the greater number of large blocks in the '4' group. But the significant statement was added, "This (pointing to the '6' group) had the most little ones though." 15A stated in connection with 4-6, "This one is bigger with big blocks. That one is bigger with little blocks."

The rôle of counting.—Counting has been supposed to play a large part in the formulation of the first arithmetic concepts of young children. In fact, counting is the first task in arithmetic given the child in a great majority of schools. Dewey claimed that the first arithmetic task to give the child is to have him count.[2] Thorndike states that simple problems in counting and measuring should form the arithmetic curriculum for the first two years.[3] This procedure tacitly assumes that

[2]Dewey, John. "Some Remarks on the Psychology of Number," *Pedagogical Seminary*, V (January, 1898), pp. 426-434.

[3]Thorndike, E. L. *The Psychology of Arithmetic* (New York: Macmillan, 1922), p. 207.

the series notion is of primary importance and that the development of quantity concepts will follow as a matter of course. The qualitative data from this study, we believe, show definitely that this older theory regarding the acquisition of number ideas is erroneous and that these data support a different hypothesis, namely: 1. The cardinal (quantity) and ordinal (series) ideas of number differentiate or develop together. 2. Counting accompanies this process of differentiation, but is not a cause of the development.

The responses from the younger children show that counting was not essential for the child to comprehend complex mathematical situations of *more* and *less*. The child understands terms relating to differences of quantity before mastering any other concept used in elementary arithmetic. (The genetic study by Decroly and Degand provides data corroborating this view.)[4] This means that an understanding of the cardinal concept must be achieved, even though undifferentiated, before the counting act can be perfectly understood. The experimenter gave Test I, Experiment A, to nine preschool children (7 girls, 2 boys) with an average age of 4 years and 10 months as a partial check of these points. These children on the average had a visual notion of three. A few may have had only a visual idea of two while one subject may have had a notion of four. There were a few in this group who could recognize and employ mathematical terms such as *most*, *both*, and *biggest*. The words *same* and *equal* (as employed in the test situation) were incomprehensible to these children. The results of this test, corroborated by the results on the older children, would indicate that mathematical concepts meaning *most* are employed by children before concepts meaning *equal*. The conclusion cannot be drawn, however, that the concept *most* may be differentiated to a high degree while the notion of *equal* remains entirely undifferentiated. Where *most* is fairly well understood, some idea of equal is present. The demand for using some idea of equality can be shown by the children's invention of the phrases: "None has the most;" "Both have the most;" "There's most on both ends."

A few cases demonstrate very strikingly the relation of counting to the development of the ordinal and cardinal number concepts. 18A understood the questions to judge which group had the *most* blocks, although his counting could not be depended upon to give him an accurate answer. In this instance both the cardinal and the ordinal con-

[4] Decroly, M., and Degand, Julia. "Observations Relatives A L'Evolution Des Notions De Quantités Continues Et Discontinues Chez L'Enfant." *Archives de Psychologie*, XII (May, 1912), pp. 81-121.

cepts of number were well comprehended. If the combination 8-9 were counted 8-8, he reported that the groups of blocks were the same. If two equal groups of blocks were counted as different, 7-7, for example, were counted as 7-6, the groups would be judged as different. In one instance 6-6 was counted 5-5 and he reported, "The same." Errors in counting were made about 75 percent of the time and yet the series idea of numbers up to ten was nearly perfect. The parents in this case purposely had given no arithmetic training to the child and the result demonstrates clearly that a pupil does not need to go through a mechanical, *rote* process of counting in order to learn the meaning of the number names. 19A was younger than 18A, but had received some training in counting. Here, likewise, the series concept of numbers up to ten was well mastered; several errors were made in counting, however, with the same result as was obtained with 18A. Numbers up to three were recognized immediately by inspection. Numbers above three were always counted.

Some insight into the counting process as a differentiation of grosser number concepts is shown by the response of 25A. The subject wished to be accurate in his responses and verified all his judgments. He was sure of himself only on numbers up to six. After indicating correctly the larger groups in 6-8, he was asked to count both groups of blocks. He replied, "I can't count all of them . . . too many." Again when 7-8 was presented he counted to six in the larger group and did not try to count above six. He said, "I can't go beyond that." He was encouraged to solve the problem, and he finally proceeded to do so in the following manner: he counted "Two, four," on the seven group and pulled the four blocks slightly to one side. "There's three," he explained, meaning that there were three beyond the first four blocks. He counted four in a similar manner in the eight group and since he recognized there were *many* beyond the four in this group he knew immediately what reply to make to the question, "Which one has the most blocks?"

The demand upon a child for a comparison of two large groups of blocks always led to the adoption on his part of some process for differentiating the groups. The evidence indicated that a group of objects meaning *many* to the child is not differentiated first into a multitude of ones. A gross differentiation precedes this more complex differentiation; the division of the group is made into more or less unequal subgroups. 25A demonstrated this fact clearly by his method of finding a difference between seven and eight blocks. The groups were 'broken up' into smaller, unequal parts. Seven became a '4' and '3'. 15A gave a similar response although under quite different circumstances. She volunteered

to count seven on Test I and reported, "There's four in back and three in front." Again on Test III she said relative to seven, "Let me count . . . there's four and three."

It is only at a later stage that a group is divided by means of counting into homogeneous parts on the basis of number. In the experimental situation the energy necessary for the child to differentiate a relatively large group of blocks by counting 1, 2, 3, and so on, was found to be tremendous. 6B, whose performance illustrates this point, depended entirely on the counting method to make her judgments. The difficulties encountered in several instances were far beyond the child although they might not have been had the groups of blocks been estimated perceptually. On one occasion, with the combination 8-8, the child became confused and although the groups of blocks were finally counted correctly, she could not remember the number in the first group after counting the second one. This problem remained unsolved. The subject was not at all accurate in counting above six and often repeated the counting three or four times.

Another subject, 9B, had difficulty in verifying judgments by counting. This subject counted carefully and in order to remember the the number of blocks in the first group would frequently repeat the last number with great emphasis. On a group of seven blocks, for example, the subject said, "1, 2, 3, 4, 5, 6, 7." Then putting both hands around the group, she said, "This is seven." She proceeded then to the second group of the combination and as soon as the counting was completed the larger group was identified. The 8-9 on Test II was missed, however, in spite of this attempt to remember the number of blocks in the first group.

VI. DISCUSSION OF RESULTS AND RELATED STUDIES

The mathematical vocabularies of school children.—Other investigators have found that the term *equal* is difficult for children to comprehend. The results of Buswell and John indicated that the words *equal* and *each* were especially difficult.[5]

Renwick found that a mathematical expression whose parts are connected by the sign of equality means to children ten to twelve years old merely an instruction to perform the process preceding the sign. When the term *equal* is introduced its meaning tends to become *same*, and to be

[5]Buswell, G. T., and John Lenore. "The Vocabulary of Arithmetic." *Supplementary Educational Monographs*, Number 38, Chicago: University of Chicago, 1931.

replaced by *alike* or is used in connection with resemblances where neither degree nor quantity is involved.[6]

The pre-school child's knowledge of arithmetic.—Douglass found that kindergarten children with few exceptions do not have accurate concepts of number above four. "Their concepts of numbers above four are so indefinite that rarely can they correctly estimate numbers above four as often as three times in five."[7] Many of these children, on the other hand, have a sufficiently adequate concept of the numbers up to to ten to enable them to estimate these numbers within a very small range of error.[8]

Decroly and Degand's subject learned two in its true significance during the twenty-eighth month; she also began to repeat mechanically, various number names: two, seven, six, and five. In the thirty-ninth month, she gave any number when asked how many three objects were. If there were a large number of objects, she did not reply with any number, but used the adverb *many*. In the fifty-fourth month the number three was known and used correctly.[9]

Methods of differentiating groups: difficulties in the counting method.—The pattern of the group of objects as a whole undoubtedly plays the leading rôle in the first stage of perceiving quantities and the relations between quantities. Unfortunately, no data except of a qualitative nature were gained in the present investigation which bear directly on this problem. It was evident in many cases, especially among the younger children, that similarity of quantity was closely allied with similarity of form. One subject, who placed one group of blocks upon the other group of blocks in Experiment A, demonstrated that the *pattern* method enabled him to solve the problem. In this case a difference of one was reduced to a difference of pattern. A correct response was given even though it was impossible for the subject to state the exact amount or number in the two quantities concerned.

Usnadze studied the methods by which pre-school children combined the objects about them into groups. Only the three-year-olds, according to his results, were unable to form groups unaided or unguided by the experimenter. The primitive mode of group formation was

[6] Renwick, E. M. "Children's Misconceptions Concerning the Symbol for Mathematical Equality," *British Journal of Educational Psychology*, II, Part I (February, 1932), pp. 173-183.

[7] Douglass, H. R. "The Development of Number Concepts in Children of Pre-School and Kindergarten Age," *Journal of Experimental Psychology*, VIII (December, 1925), p. 455.

[8] *Ibid.* p. 467.

[9] Decroly, M., and Degand, Julia. *Op. Cit.*

made according to spatial relations of the subject to the material and it was only at a later stage that the characteristics of the material itself were taken into consideration. Usnadze's results show very strikingly the difficulty of the number concept. "The most difficult characteristics for grouping were found to be numbers of dots and colors. The independent use of numbers for grouping without guidance by the experimenter was first noticed with six-year-old children . . ."[10]

An experiment by Luria not only shows the difficulty of number concepts, but also gives some evidence in support of the principle advanced here that children tend to differentiate large groups of objects into unequal subgroups before employing counting by ones to differentiate these groups.[11]

The maturation of mathematical concepts has not been thought to play a very important rôle in learning to count after the number names have once been acquired. Buckingham and MacLatchy, for example, reached the conclusion that when the child counts ten or more objects *fairly accurately*, he has gained considerable knowledge about these higher numbers.[12] Judd, on the other hand, found that counting in children really develops quite slowly and involves a complex, mature comprehension of number meaning. With the counting of flashes of lights, fifth grade pupils were far less capable than adults. The great amount of improvement found in the performance of children as one proceeds from the second or third to the sixth grades is rather significant in view of the fact that special instruction or special drill in counting is not given in these grades. The importance of realizing the difficulties involved are summed up in the following statement, "If the relatively simple process of counting is as complex as it has been discovered to be, if it matures as slowly as the tables of this monograph show, and if an understanding and appreciation of the meaning of counting are as mature forms of consciousness as the discussion of the last few pages indicates, how much more complex must be the mastery of number combinations and the later phases of arithmetic?"[13]

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[10]Usnadze, D. "Studies in Grouping by Pre-School Children," Reviewed in *Psychological Abstracts*, IV (October, 1930), p. 462.

[11]Luria, A. R. "Experimental Psychology and the Development of the Child," Reviewed in *Psychological Abstracts*, IV (January, 1930), p. 46.

[12]Buckingham, B. R., and MacLatchy, J. "The Number Abilities of Children When They Enter Grade One," *Twenty-Ninth Yearbook, National Society for the Study of Education* (Bloomington, Illinois: Public School Publishing Company, 1930), Part II, Chap. IV, pp. 473-524.

[13]Judd, C. H. "Psychological Analysis of the Fundamentals of Arithmetic," *Supplementary Educational Monographs*, No. 32 (Chicago: University of Chicago, 1927), p. 52.

Criticism of the conclusion reached by Buckingham and MacLatchy.—There are several investigators whose conclusions have differed from those maintained here.[14] We wish, however, to discuss only one as typical of this group, namely, that of Buckingham and MacLatchy. These educators report a rather extensive study concerning the number 'abilities' of children when they enter grade one. They concluded that "if we may regard the children in this study as typical. . . it seems probable that at least one-half and possibly two-thirds of the pupils who enter the first grade can successfully negotiate by means of objects any of the one hundred basic facts of addition."[15]

Three objections can be raised to the conclusions of Buckingham and MacLatchy. First, children of this age *are not accurate* in their responses to arithmetic problems involving numbers over five, but are quite clever at 'guessing' the answers. This is shown rather strikingly in Buckingham and MacLatchy's own results wherever more than one trial was allowed for gaining the correct response (except counting problems, see particularly Tables VI and VIII of their report). Buckingham and MacLatchy, even though they have insisted on complete accuracy as the criterion of success, are inclined to give too much credit to the children who attained 'partial' success. These children, they believe, are 'on the way.' (See, for example, the interpretation given to the results of the reproducing of five.[16] The fundamental difference between the interpretation of these authors and the one maintained here becomes most evident on this very point. These authors hold that partial knowledge of the numbers five to ten indicates that a pupil with only a little additional experience will stabilize his action sufficiently for a perfect response. The theory adhered to here supposes that the 'way' to perfect knowledge of the higher numbers may be most difficult for not a few of these pupils and that, furthermore, the mastery of number is a most complex process.

Second, the tests of these investigators were measures chiefly of abstract responses. It should be recognized that such processes as counting when isolated from the whole arithmetic situation become artificial and

[14] Four of the most important of these studies are as follows:
Buckingham, B. R., and MacLatchy, J. *Op. Cit.*
Washburne, C. "When Should We Teach Mathematics? A Committee of Seven Investigation," *Elementary School Journal*, XXVIII (May, 1928), pp. 659-665.
Washburne, C. "Mental Age and the Arithmetic Curriculum: A Survey of the Committee of Seven Grade Placement Investigations to Date," *Journal of Educational Research*, XXIII (March, 1931), pp. 210-231.
Woody, C. "The Arithmetic Background of Young Children," *Journal of Educational Research*, XXIV (October, 1931), pp. 188-201.

[15] Buckingham, B. R., and MacLatchy, J. *Op. Cit.*, p. 506.

[16] *Ibid.* p. 498.

unimportant from the point of view of the child. In other words, a counting method *in itself* will reveal little concerning the child's true ideas about quantities and about the number series. A statement by Buckingham and MacLatchy concerning their own report is of importance in this connection. "There is nothing here about comparison, nothing about a quantitative vocabulary. . . ."[17]

Third, Buckingham and MacLatchy assume that since first grade children have a good background of number ideas they are ready for formal instruction in this subject. It is true that first grade pupils perform about as well on certain tests as do second graders or even third graders. It was found by Woody, Douglass, and also in the present study, as well as by Buckingham and MacLatchy, that young children possess considerable arithmetical information. But the conclusion does not follow from these results that *any pupil* from four to eight years of age is competent to receive formal arithmetic instruction as it is now presented in many elementary schools. It is known that difficulties in the comparatively simple tasks of processing have been experienced by pupils as far advanced as the eighth grade and in some instances even by students in college. It is our contention that the investigator needs to follow the mastery of number concepts as a continuous, maturation and differentiation process, of which counting is secondary, and to interpret his findings in relation to this larger picture of mental growth. No accurate prediction of the development of number ideas will be possible except as this more inclusive problem is taken into consideration.

VII. CONCLUSION

1. The results lead to the conclusion that the child's first concept of number is a *manyness* from which the quantity and serial aspects of number differentiate. Data were presented to show that the differentiation is a gradual process which, at the seven-year-old level and beyond does not reach the adult's conception of the cardinal and the ordinal ideas of number.

2. The present study substantiates the hypothesis that the cardinal and ordinal number concepts develop simultaneously. Ability to count, in itself, is not a reliable measure of this development.

3. The child, four and a half to five years of age, readily understands the terms *most*, *both*, and *biggest*. Words denoting same and *equal* are not comprehended. The child can compare groups of blocks up to ten with remarkable accuracy, although he has a visual notion only of three or perhaps of four.

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[17] *Ibid.* p. 484.

4. The seven-year-old child uses such terms as *many*, *most*, and *more*. The words *same* and *equal* are not fully comprehended. Counting by ones is a difficult method for differentiating groups and is not accurate above five. The child will form subgroups first which have unequal value mathematically. At a later *stage* in the differentia-process, counting by ones is employed.

5. It is not likely (as many have maintained) that the first grade or second grade pupil will be mature enough to *master completely* and *understand* isolated addition and subtraction facts. Formal work such as drill over these arithmetic facts should be discouraged. The observations indicate that the initial training in arithmetic should be undertaken with the use of concrete materials.

THE COMPARATIVE WORTH OF TWO DIAGNOSTIC TECHNIQUES IN ARITHMETIC

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DIAGNOSTIC procedures in arithmetic may be roughly grouped into two classes: (1) those (called PI in table headings) which require intimate contacts with individual children (the personal interview and the case method), and (2) those (called AT) which depend upon the analysis of written records of pupils' performances (test papers, for example). Of the two groups of procedures, the second has been much more popular. A tabulation of studies published through the year 1930 and dealing with "pupils' processes" and "pupils' difficulties" in arithmetic reveals the ratio to be about 3:2 in favor of the second group. This ratio, however, is probably very much too low to describe the situation with respect to practical classroom diagnosis. Here the ratio may mount as high as 3:1 or even 5:1.

The greater popularity of the second group of diagnostic procedures is easily explained: (1) the skills under investigation can be organized into tests which may then be administered by almost any one; (2) records can be obtained from whole classes in the time that would be sufficient for but a single individual interview; (3) the records obtained are permanent; and (4) the papers can be analyzed at any time, as opportunity permits.

Nevertheless, real as they are, these advantages for diagnosis by the analysis of test papers should be recognized for what they are, namely, the advantages of convenience. On this account it is not altogether unreasonable to inquire whether the case may be safely allowed to rest upon mere convenience. If the facts obtained through the analysis of test papers are not to be trusted, there remains little to be said for this type of diagnostic procedure.

Those who have tried to study children's processes more immediately, by observing children at work and by questioning them concerning their methods, are quick to criticize the results secured by analyzing papers. They insist: (1) that the categories arrived at are adult categories,—descriptions of the mature mind at work rather than of the child's

[1]The junior author is responsible for all the computations in this report.

mind at work; (2) that the method is unreliable, for the reason that but few responses can be identified and classified with certainty; and (3) that the total pattern of the child's behavior is artificially broken down, and the relationships between the child's thought processes are lost. The advocates of the personal interview acknowledge the inconvenience of their diagnostic procedure, but they maintain that procedures less direct than theirs are fraught with danger of serious error.

In their turn, those who favor diagnosis by the analysis of test papers experience no difficulty in detecting weaknesses in the personal interview. (1) The latter procedure is at the best laborious and time-consuming. Consequently it is not likely to be generally employed or to yield facts of general application. (2) The method is overly "subjective,"—the interviewer has to trust too much to his judgment. (3) The interview cannot be standardized. Hence, variation is inevitable from interview to interview, and from interviewer to interviewer. (4) Pupils' responses are apt to be distorted. Quite without intent, the interviewer is apt to suggest the answer he wants or to supply cues that unduly affect children's reactions.

PURPOSE AND METHOD OF INVESTIGATION

The purpose of the investigation summarized in this paper was to secure data on the comparative worth of the two kinds of diagnostic procedure under discussion. The data to be reported do not warrant a final answer to the question, but at least they represent a beginning on the basis of quantitative facts rather than of argument.

In brief, the method was as follows. Each child was given a copy of the test and was asked to write his work and his answers in the ample space provided for that purpose. Furthermore, he was directed to "think out loud" as he worked. The interviewer kept another copy of the test before him and entered the child's statements, as well as significant items of his behavior not reported by the child. The result was two records for each child: (1) a test paper which could later be analyzed in the usual manner and (2) an interview report on his processes. The two records were then available for a number of comparisons to be described shortly.

At this point one limitation of the study must be recognized. In classroom diagnosis the interviewer may carry his questions concerning the child's processes to considerable length in order to secure ultimate explanations. In this investigation, however, this practice was inadvisable,—to have pursued the inquiry far beyond the words spoken spontaneously by the pupil would have ruined both the interview report and the child's paper as a record of normal behavior. Each subject was there-

fore urged to talk openly, honestly, and completely, and that was all. The consequence was, it is believed, a typical test paper, but by no means so typical an interview report.

The subjects of the investigation were all the children in the fifth grades of seven schools in Nashville, Tennessee. Of the 250 subjects available, usable records were obtained from 245. The seven schools represented a normal sample of the schools in that area. They were chosen from both urban and suburban districts, and from economic circumstances of the poorest at one extreme to the best at the other extreme.

The test used was modified from the Brueckner Diagnostic Test in the Addition of Fractions. As revised, it comprised fifteen examples which fall into three major types. In the first five examples the common denominator occurs in both the fractions to be added; in Examples 6-9 the common denominator occurs in only one of the fractions; and in Examples 10-15 the common denominator must be found.

TEST USED IN THE INVESTIGATION

ADD:

(1)	$\frac{1}{5}$	(2)	$\frac{1}{8}$	(3)	$\frac{2}{7}$	(4)	$\frac{4}{5}$	(5)	$\frac{4}{6}$
	$\frac{2}{5}$		$\frac{5}{8}$		$\frac{5}{7}$		$\frac{2}{5}$		$\frac{4}{6}$
(6)	$\frac{1}{2}$	(7)	$\frac{1}{2}$	(8)	$\frac{2}{3}$	(9)	$\frac{4}{8}$	(10)	$\frac{3}{8}$
	$\frac{1}{4}$		$\frac{2}{8}$		$\frac{3}{6}$		$\frac{3}{4}$		$\frac{1}{12}$
(11)	$\frac{1}{5}$	(12)	$\frac{2}{4}$	(13)	$\frac{3}{7}$	(14)	$\frac{3}{6}$	(15)	$\frac{7}{9}$
	$\frac{2}{4}$		$\frac{7}{11}$		$\frac{3}{4}$		$\frac{4}{5}$		$\frac{5}{6}$

The interviewers were five in number. In addition to the senior author, they were Mrs. Flora M. Gillentine, Miss Myra Hodges, Miss Virginia Howard, and Mr. J. G. Garrett, graduate students enrolled at the time in a course entitled "Research in Arithmetic." Each interviewer observed three interviews given by the senior author, and then himself

interviewed three other children under critical observation. None of the subjects of these preliminary interviews was included in the investigation proper.

Before the test was presented to the child, he was given practice in reporting his processes until it was clear he knew what was expected of him. The interviews required approximately twenty minutes each, or a total of more than seventy hours, distributed over a period of approximately two months. All of the children had been instructed from the same text and according to the same course of study, and had completed the topic, addition of common fractions, not less than two and one-half months or more than three and one-half months before they were tested.

In all, fifty-five categories of "faulty procedures" or "faults" were identified. These categories were derived from the interview records, a fact which constitutes another limitation to the conclusions which may be drawn. Manifestly, a common basis for comparison was needed. If the categories had been derived independently both from the interviews and from the test papers, no such common basis would have been found. It was necessary therefore to take the categories from either the interview records or the test papers, and the former were chosen.

For the purposes of this report the exact nature of these categories is hardly relevant. However, descriptions of the ten commonest faults will be found in Column 1 of Table I. These ten faults were responsible for eighty percent of the total number identified.

RESULTS OF GROSS ANALYSES

In the rest of this article the letters PI will be used to designate the personal interview and the data obtained thereby, and the letters AT to designate the analyses of test papers and the corresponding data.

Some of the more obvious types of comparison are summarized in Columns 2 to 5 of Table I. If one compares the frequency with which Fault Type Id appeared in the work of the 245 children, the figures are 177 for PI and 171 for AT. The discrepancy is far from large. If now one continues his comparison of the two diagnostic procedures to include the other nine commoner faults and the totals for all 55 faults, again similarity, rather than marked difference, is the rule. Thus for the ten faults, the total frequencies are 3047 as compared with 3037, for PI and AT respectively, and the totals for all 55 faults are 3735 as compared with 3718.

Columns 4 and 5 of Table I are based upon the number of pupils who used each given type of fault, regardless of the frequency with which they employed it. A comparison of the pairs of figures in these two columns reveals again fairly close agreement. The conclusion seems to be that

TABLE I

NUMBER OF USES AND OF USERS OF TEN COMMONEST FAULTS AS REVEALED BY PI AND BY AT

	Type of Fault	Total Uses		Total Users	
		PI	AT	PI	AT
	(1)	(2)	(3)	(4)	(5)
Id	Add denominators to secure common denominator	177	171	21	17
IIc	Used complicated form to find common denominator when latter was already given	91	98	17	17
IId	Used common denominator unnecessarily large	430	407	134	129
IIIg	Added denominators to secure denominator of the answer	141	119	10	12
IVa	Made computational errors in operating on numerators	78	82	60	49
IVc	Added numerators without change	277	291	31	26
Vb	Failed to reduce proper fraction	986	995	195	189
VIb	Failed to reduce improper fraction	509	504	103	103
VIf	Failed to recognize unity in fractions like $\frac{7}{7}, \frac{4}{4}$, etc.	83	81	74	68
VIIc	Needlessly rewrote example in horizontal form	275	289	20	18
Total, 10 Faults		3047	3037	667	628
Total, 55 Faults		3735	3718	940	880

PI and AT are about equally effective in identifying the total number of various faults when as many as 245 pupils are involved.[2]

Likewise, there seems to be little to choose between PI and AT when

TABLE II

TOTAL NUMBER OF FAULTS PER EXAMPLE, PI VS. AT

Example	Number of Faults according to	
	PI	AT
1	164	163
2	242	232
3	197	194
4	230	228
5	318	316
6	180	176
7	224	220
8	195	194
9	284	294
10	192	188
11	231	234
12	327	335
13	233	225
14	316	327
15	402	392

[2]Attention has been called to the fact that items such as pupils' difficulties in a school subject may be classified (1) in terms of the number of times these difficulties occur (frequency) or (2) in terms of the number of individuals who manifest these difficulties (universality). For instructional purposes the significance of a high frequency for a given kind of difficulty will vary, depending upon whether that frequency is due to repeated errors in the case of a few pupils or to general occurrence. It is of interest therefore to note that frequency and universality of faults correlated .92 for PI and .87 for AT for the 245 subjects in this study.

a comparison is made of number of faults per example. According to Table II a total of 163 faults were found in the 245 attempts to solve Example 1 when the analysis was made from the interview records, as compared with 164 when the analysis was made from the tests. The other pairs of figures in Table II are read in similar fashion, and they again reveal agreement.

Table III summarizes a number of other gross comparisons between PI and AT. Here the coefficient of correlation is employed to measure the degree of agreement. In some instances r is used, in others, ρ. The Rank Method of computing correlation was used instead of the Pearson Product-Moments Method whenever the data revealed wide gaps in the scale, and wherever the number of paired cases was small.

TABLE III

CORRELATIONS FOR VARIOUS UNITS OF COMPARISON BETWEEN PI AND AT

Scorer		Unit of Comparison	PI vs. AT
x vs. y (245 pairs of records)	(1)	No. examples missed per paper (Scale = 0–15)	$r = .87 \pm .002$
	(2)	No. faults per paper (Scale = 0–47)	$r = .87 \pm .002$
	(3)	Total No. faults per example (Scale = 1–9–400)	$\rho = .93 \pm .01$
	(4)	Total No. faults per type of fault (Scale = 1–335)	$\rho = .71 \pm .05$
	(5)	Total No. users per type of fault (Scale = 1–195)	$\rho = .69 \pm .04$
x vs. y (50 pairs of records)	(6)	No. faults per type per paper (Scale = 0–15)	$r = .55 \pm .04$
x vs. x (40 pairs of records)	(7)	No. examples missed per paper (Scale = 0–15)	$r = .83 \pm .02$
	(8)	No. faults per paper (Scale = 0–35)	$r = .81 \pm .04$
	(9)	No. faults per type (Scale = 0–15)	$r = .55 \pm .09$
	(10)	No. faults per type per paper (Scale = 0–15)	$r = .42 \pm .05$

The first six coefficients of correlation represent the results of comparing 245 pairs of records, when interview records were analyzed by one individual and test papers were analyzed by another. The last four coefficients show the degree of correlation between 40 pairs of records, when both PI and AT were the work of the same individual, AT being made independently of PI, and some time later.

The coefficients for Comparisons 1, 2, 3, 7, and possibly 8 show little difference between the results of PI and AT. Here the diagnosis involved merely the distinction between correct and incorrect computation (Comparisons 1 and 7) and the total number of faults per paper (Comparisons 2 and 8) and per example (Comparison 3).

The coefficients for Comparisons 4, 5, 6, 9, and 10 disclose much less agreement between the results of the two types of diagnosis. Apparently when one passes from the crude bases either of total examples missed per paper or of total faults per paper or per example to a consideration of types of fault as such, it does make a difference whether one proceeds by PI or by AT.

RESULTS OF CLOSER COMPARISON

The consistency of diagnosis by PI and by AT is compared in Tables IV and V in terms of percentage. Column 4 of Table IV shows that the work for only about three out of ten pupils on the test as a whole was analyzed by PI and by AT in exactly the same manner, while eight out of ten examples were consistently analyzed. These figures are high largely for the reason that the percentages include the examples in which no errors were made and in which none of the faulty procedures appeared. When the comparison was restricted to examples in which errors appeared, the agreement fell to 68.6 percent. Approximately the same degree of relationship held for the disposition of the 3735 separate faults.

For the practical problem of classroom diagnosis it is the figures 68.6 percent and 74.4 percent which are significant, for such diagnosis is concerned with mistakes and with faults which interfere with successful achievement.

TABLE IV

CONSISTENCY OF ANALYSIS BY PI AND AT

Unit of Comparison	Total No. Involved	No. Consistently Analyzed	Percent Consistently Analyzed
(1)	(2)	(3)	(4)
All papers	245	72	29.4%
All examples	3675	2959	80.5
Examples missed	2210	1518	68.6
No. faults	3735	2779	74.4

The basis of comparison in Table V is the separate examples in the test. In the case of Example 1, 84.1 percent of the 245 comparisons were in agreement. That is to say, the same disposition of faults was made both by PI and by AT in the case of 84.1 percent of the 245 attempts to solve Example 1. The median for the fifteen examples is 82.0 percent. This figure is again made high by the inclusion of papers which were almost necessarily diagnosed alike because of the absence of errors or of faulty procedures. On this account, the figures in Column 3 are important. In the case of Example 1, PI and AT agreed in classifying alike only 52.7 percent of the faults when those papers were considered in which the two diagnoses were not exactly alike. The median for the fifteen examples is 51.5 percent. Another way of expressing the relationship represented in this figure is to say that PI and AT agreed in classifying only half of the faults when there was any disagreement at all.

Another way of checking the consistency of PI and AT in classifying faults was to compare random samples of paired interview records and

TABLE V

PERCENTAGE OF CONSISTENCY OF PI AND AT IN CLASSIFYING THE FAULTS IN THE 15 EXAMPLES PER EXAMPLE

Example	Percent of Papers n Agreement	Percent Agreement on Examples Not Wholly in Agreement as to Diagnosis
(1)	(2)	(3)
1	84.1%	52.7%
2	85.7	56.6
3	82.8	62.5
4	80.8	57.3
5	73.4	58.7
6	80.8	51.5
7	83.2	48.9
8	83.7	60.3
9	82.4	51.0
10	79.6	48.1
11	82.0	50.0
12	82.0	46.1
13	80.8	50.0
14	78.8	50.0
15	71.0	56.9
Median	82.0%	51.5%

test papers. The two types of record for fifty children constituted the first sample. The number of each kind of fault was determined for each record independently. In all, the fifty pairs of papers yielded 179 pairs of measures, from which an r of $.55 \pm .04$ was obtained. With another random sample of forty pairs of records, which yielded 114 pairs of measures, r was found to be $.42 \pm .05$ when the same scorer analyzed the papers by each of the two methods.

The correlation coefficients reported in Comparisons 4, 5, 6, 9, and 10 of Table III and in the foregoing paragraph, as well as the low percentages in Column 3 of Table V, make it clear that the two diagnostic techniques, PI and AT, are not consistent when applied to particular types of fault. It becomes a matter of some importance then to know which of the two procedures is more reliable.

COMPARATIVE RELIABILITY OF PI AND AT WITH RESPECT TO PARTICULAR FAULTS

Table VI summarizes certain comparisons of PI and AT. In these comparisons forty interview records were analyzed twice by the same person, and thirty test papers twice by the same person. The important figures are those in Columns 6 and 7. Note that when total papers are considered, the two diagnoses by PI were 45 percent consistent, by AT, 30 percent consistent; that the corresponding figures were respectively 79.8 percent and 67.2 percent when the comparison was restricted to examples with faults; that the respective figures were 83.6 percent and 63.7 percent when the unit of comparison was the number of faults per paper. In every instance PI proved to be noticeably more consistent, or more reliable.

TABLE VI

PERCENT OF CONSISTENCY BETWEEN THE FIRST AND SECOND ANALYSIS OF THE SAME PAPER BY THE SAME SCORER

Unit of Comparison	Total Number Involved		Number Consistent		Percent Consistent	
	PI	AT	PI	AT	PI	AT
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Total number papers...	40	30	18	9	45	30
Number examples with faults.....	654	518	520	349	79.8	67.2
Number faults.....	898	919	751	586	83.6	63.7

Further measures of the comparative reliability of the two diagnostic procedures were obtained by the following correlational studies:

(1) The same individual analyzed the interview records of forty children twice. The coefficients of correlation which resulted are listed in Part A of Table VII, Column 3 under the heading PI vs. PI.

(2) The same individual analyzed thirty test papers twice. The coefficients for these comparisons are listed in Part A of the table, Column 4, AT vs. AT.

(3) The same forty personal interviews were analyzed independently by two different individuals (Part B, Column 3).

(4) The same forty test papers were analyzed by three different individuals (x, y, z) (Part B, Column 4).

It is sufficient to call attention to the coefficients themselves (Table VII). Regardless of the unit of comparison (examples missed per paper, total faults per paper, total faults per type, and number of faults per type per paper), and regardless of the fact as to whether the two analyses were made by the same or by different individuals, PI proved to be more reliable. The differences in reliability are probably insignificant for Comparisons 1, 2, 5, and 6, in all of which cases, however, the kind of diagnosis involved is rather superficial. In the case of the kinds of diagnosis which are more important for the direction of learning for the individual pupil (Comparisons 3, 4, and 7) the difference in reliability is marked. In these cases PI is much to be preferred to AT. These conclusions, it will be observed, agree with others announced earlier in this paper.[3]

[3]It must be borne in mind that these coefficients as measures of the reliability of the two procedures are spuriously low because they involve only the correlation of faults or of examples involving faults. If the examples in which no faults appeared had been included in the correlations, the coefficients would have risen considerably.

TABLE VII

RELIABILITY COEFFICIENTS* FOR PI AND AT

Scorer (1)	Unit of Comparison (2)	Method: PI vs. PI (3)	Method: AT vs. AT (4)
Part A: x vs. x	(1) Examples missed per paper	.93 ± .02	.91 ± .03
	(2) Total faults per paper	.92 ± .02	.85 ± .03
	(3) Total No. faults per type	(ρ) .78 ± .05	(ρ) .59 ± .08
	(4) No. faults per type per paper	.83 ± .02	.73 ± .03
Part B: x vs. y x vs. z	(5) Examples missed per paper	.94 ± .02	.89 ± .04
	(6) Total faults per paper	.87 ± .04	.83 ± .04
	(7) No. faults per type per paper	.68 ± .05	x vs. y .32 ± .05 x vs. z .47 ± .04

*r, unless otherwise indicated.

Table VIII contains a summary of all the correlational data obtained. The reader who has pursued the inquiry thus far should be able to interpret the table without explanation. Column 5 repeats the data from Table III.

TABLE VIII

SUMMARY OF THE CORRELATIONAL DATA*

Scorer (1)	Unit of Comparison (2)	Method of Comparing: PI vs. PI (3)	AT vs. AT (4)	PI vs. AT (5)
x vs. x	(1) No. examples missed per paper	.93 ± .02	.91 ± .03	.89 ± .02
	(2) No. faults per paper	.92 ± .02	.85 ± .03	.81 ± .04
	(3) No. faults per type	(ρ) .78 ± .05	(ρ) .59 ± .08	.55 ± .09
	(4) No. faults per type per paper	.83 ± .02	.73 ± .03	.42 ± .05
x vs. y or x vs. z	(5) No. examples missed per paper	.94 ± .02	.89 ± .04	.87 ± .002
	(6) No. faults per paper	.87 ± .04	.83 ± .04	.87 ± .002
	(7) No. faults per type			(ρ) .71 ± .05
	(8) No. faults per type per paper	.68 ± .05	x vs. y .32 ± .05 x vs. z .47 ± .04	.55 ± .04
	(9) Total No. users per type of fault			(ρ) .69 ± .04
	(10) Total No. faults per example			(ρ) .98 ± .01

*r, unless otherwise indicated.

COMPARATIVE VALIDITY OF PI AND AT

Since reliability contributes directly to validity, the greater reliability of PI argues for its greater validity in the detailed diagnosis of

faulty procedures. This reason is statistical in character. There is another type of evidence which may, or may not, be more convincing, namely, the nature of the records used in PI and in AT.

Below are given three samples of the two kinds of record. The uncertainties of classifying the faulty procedures from the test paper are apparent. Without the notes on what the child was thinking, the identification of the faults here illustrated becomes little more than guesswork,—even when, as may or may not happen, peculiar faults appear several times in the same paper. Again, from the standpoint of remedial instruction the information to be had from the interview would appear to be more valuable than that gained from the test paper alone.

SAMPLES OF RECORDS

Example	Pupil's Work	Interview Record
$\frac{4}{5}$ $\frac{2}{5}$	$\frac{4}{5}$ $\frac{2}{5}$ $\frac{5}{6}$	"When you add fractions, you change the numbers around, so you write $\frac{5}{6}$ instead of $\frac{6}{5}$ for the answer."
$\frac{1}{8}$ $\frac{5}{8}$	$\frac{1}{8}$ $\frac{5}{8}$ $\frac{6}{8}=1\frac{2}{8}$	Last step: "8 won't go into 6, so you divide 6 into 8, and that leaves $\frac{2}{8}$ over."
$\frac{4}{8}$ $\frac{3}{4}$	$\frac{4}{8}$ $\frac{3}{4}$ $\frac{10}{8}=2\frac{1}{8}$	Last step: "Take away 8 from 10. That leaves 2, and you write the 1 you got over the 8."

In this connection it should be repeated again that PI was placed under a severe handicap by the conditions of the study. Much more revealing evidence of the nature of these pupils' difficulties would have been secured had it been possible to press the questioning as far as is feasible in ordinary classroom diagnosis. Nevertheless, even in this study there were collected many illuminating examples of pupil difficulties,—difficulties open only to the intimate relations of the interview, in which the language of the child's thought is open to observation.[4]

What facts obtainable by AT could possibly provide the teacher as significant instructional clues as do the following illustrations chosen from interview records? In each case a child was describing his processes in finding a common denominator.

a. "4 is the common denominator because 4 will go into 8 and 12."
b. "18 is in 6 tables 3 times, and in 9 tables 2 times."
c. "Find something to go into both."
d. "I times 'em."
e. "I multiplied them together when I couldn't think."
f. (For $\frac{3}{6}$ and $\frac{4}{5}$) "Ain't no number will exactly divide 6 and 5."
g. (For $\frac{1}{2}$ and $\frac{2}{4}$) "Two is contained exactly into 4, and you drop the 2 out."

CONCLUSIONS

The degree to which the conclusions drawn from this study are of general application is limited in at least four ways:

(1) The investigation was restricted to but a single function, and that a very narrow one, namely, the addition of proper fractions. It may be that had the skills covered by the test included a wider area of arithmetical computation, different results would have been obtained.

(2) PI was used under unfavorable circumstances,—necessarily so because of the nature of the investigation. Nevertheless, since the showing for PI is superior to that for AT except in the more superficial types of diagnosis, it may be inferred that the advantage for PI is actually larger than that found in this study.

(3) The categories for the comparisons were derived from interviews. Some of the categories found by PI would not have been detected at all by AT.

(4) All comparisons involved 245 children, or random selections of that number. On this account the findings are of the kind

[4] In this connection see also: Burge, Lofton V. "Interview Technique as a Means of Diagnosing," *Journal of Educational Research*, XXVII (February, 1934), pp. 422-429.

which are sought through classroom diagnosis. If the number of children in particular classes, numbering 20 to 36, had been used, the differences between PI and AT would probably have varied from these reported.

Within the limitations noted, the conclusions seem to be justified that PI and AT are equally satisfactory only for the grosser types of diagnosis (total examples missed, total number of faults detected, total number of faults of particular types). For these types of diagnosis AT is to be preferred because of its convenience-values. When, however, diagnosis is that of the processes and difficulties of individual children PI is both more reliable and more valid.

TRANSFER OF KNOWLEDGE OF MULTIPLICATION FACTS TO THEIR USE IN LONG DIVISION

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IN a previous study by the writer,[1] it was found that errors of combinations had the greatest frequency of any type of error made in long division when the divisor is a one-figure number. In this same study, it was shown that many errors in division were caused by lapses in attention. From an observation of many errors on the test papers from students in Grades V-XV, inclusive, (Grades XII, XIV, and XV are in normal school), the writer concluded that frequent incorrect products in multiplication resulted from the fact that multiplication was used in division. In other words, if a response had been made to a multiplication fact isolated from the division process, the pupil would have been able to give the correct product. However, if the same fact occurred in the division process, there was a possibility of an incorrect product being given to the multiplication fact. This hypothesis can be substantiated by an illustration,

$$\begin{array}{r} 7 \\ \hline 7)38 \\ 35 \end{array}$$

. In the given illustration, the pupil estimated the number of 7's in 38 and very probably 5 was the estimated figure because the product of 7 and 5 was written in the partial dividend. However, due to a temporary lapse the wrong factor of the product was written in the quotient. A multiplication error resulted because the product of divisor and quotient did not equal the partial product. It is almost certain that a multiplication error for the combination, 7 x 5, would not have resulted if the fact had been isolated from division. As a result, one can never be sure of complete transfer of ability to give the multiplication products when the facts are isolated to the ability to use the same facts in long-division.

Knight challenged the idea of ready transfer of ability to multiply in multiplication to ability to multiply in long-division.

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[1]Grossnickle, Foster E. "Errors and Questionable Habits of Work in Long Division with a One-Figure Divisor," *Journal of Educational Research*, XXIX (January, 1936), pp. 355-368.

"In multiplication, the drift of the numbers the child writes is to the left; in division it is to the right—

In multiplication, the multiplying is an uninterrupted process; in division, multiplication is sandwiched in between quotient estimation, subtraction, and bringing down of figures from the dividend. The multiplier in multiplication is present from the beginning; in division the pupil writes the digits of his multiplier as he goes along. The habits built up in multiplication, as multiplication, are enough to carry multiplication in division, only when considerable adaptability can be assumed in the learner."[2]

To see the amount of transfer between ability to respond to the multiplication facts when isolated and ability to use the facts in long-division, two separate tests were given to 1075 students in Grades V-XV, inclusive. The first test consisted of the 50 multiplication facts, arranged in an insequential order, which are used in the division test as reported in a previous study.[3] The multiplication facts were written in the equation form. The test was divided into halves. The second half consisted of the same facts as in the first half, but the arrangement of the facts on the second part was reversed from that on the first part. The student was instructed to write the products in the first half of the test before he began the second half. Each fact was given twice to determine if an incorrect response was due to chance or to a faulty knowledge of the fact. If a fact was missed twice, the resulting error was assumed to be consistent and not due to chance. Although each fact should have occurred at least three times on the test,[4] it was not feasible from the administrative standpoint to include each fact more than twice. When the student had finished the multiplication test, he solved the division test already cited.[5] With the exception of a few cases in Grades XIV and XV, none of the subjects had taken the division test before. The multiplication test was new to all of the subjects tested.

When the errors were studied in the division test, only errors of multiplication were recorded. The relationship between the errors of multiplication on the multiplication test and the errors of multiplication on the division test is shown in Table I. It must be remembered

[2]Knight, F. B. "Some Consideration of Method," *Twenty-Ninth Yearbook of the National Society for the Study of Education* (Bloomington, Illinois: Public School Publishing Co., 1930), p. 173.

[3]Grossnickle, Foster E. "Reliability of Diagnosis of Certain Types of Errors in Long Division with a One-Figure Divisor," *Journal of Experimental Education*, IV (September, 1935), pp. 7-16.

[4]Brueckner, Leo J., and Ewell, Mary. "Reliability of Diagnosis of Error in Multiplication of Fractions," *Journal of Educational Research*, XXVI (November, 1932), p. 185.

[5]Grossnickle, Foster E. *Ibid.*

that an error as reported in the following tables refers to an error of multiplication.

Table I shows some very startling facts about transfer of ability to multiply in multiplication to the use of multiplication in division. Items 2, 3, 4, and 5 are very significant from the standpoint of reliability of diagnosis. In Item 2, there were 68 examples in multiplication which were missed twice, but only 9 of those were used incorrectly in division. Item 3 shows that 76 facts were missed two or more times in division, but there were only 7 of these which were missed in multiplication. Furthermore, from this group of 76 facts in division, two of them in Grade XI were used incorrectly 5 times each, but in the multiplication test these facts were given correctly both times. When the student gave an incorrect response twice to a multiplication fact in multiplication and then used it correctly in division, there were two possible reasons for this phenomenon. First, a fact must occur more than twice on a test before reliable results may be obtained concerning a pupil's knowledge of the fact. Second, a pupil's knowledge of a fact is not constant at all periods of consciousness. There was a lapse of a few minutes between the time the pupil responded to the multiplication facts and the time he used the facts in division. During the interval a new association may have been made which enabled him to give the correct product to a fact used in division even though the same fact was missed twice in multiplication. Knight and Ford[6] found that a pupil's ability to multiply wavered in the same example.

The most striking data are given in Items 12, 13, and 14. Item 12 shows that there were 371 incorrect responses to the significant multiplication facts (those not including zero). Item 13 shows that there were 764 incorrect multiplication facts given in the division test. The latter number is more than twice the amount of the former. Including duplications, there were 1135 incorrect responses given to multiplication facts in the two tests. Naturally, one should predict that a large percentage of those errors would be duplicates. However, Item 14 shows that such an assumption is fallacious. From that group of 1135 errors there were only 22 which were common to both. Then, there were less than two percent of the total number of errors which were common to both processes.

Since there were so many more errors of multiplication in division than in multiplication, one may infer that when the pupil has difficulty

[6]Knight, F. B., and Ford, E. "Temporary Lapses in Ability and Error in Arithmetic," *Elementary School Journal*, XXXII (October, 1931), pp. 111-24.

TABLE I

RELATION BETWEEN MULTIPLICATION-ERRORS FOR SIGNIFICANT FACTS IN MULTIPLICATION AND MULTIPLICATION-ERRORS IN DIVISION FOR GRADES V-XV

	ITEMS	Grades V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	Total for all Grades
1.	Number of errorless papers in both multiplication and division	25	37	51	50	22	37	31	47	55	68	50	473
2.	Errors in multiplication, that occurred twice on each test paper	0	7	1	4	10	3	19	7	7	6	4	68
3.	Errors in division that occurred two or more times on each test paper	4	9	7	6	7	6	15	5	8	5	4	76
4.	Number of errors from Group 2 that were found in division	0	0	1	1	0	1	4	1	0	1	0	9
5.	Number of errors from Group 3 that were found in multiplication	0	0	1	1	0	0	2	1	1	1	0	7
6.	Number of multiplication facts missed once per test paper	9	13	11	21	18	13	14	32	31	39	34	235
7.	Number from Group 6 that occurred in division	0	0	0	1	0	0	1	2	1	1	1	7
8.	Number of multiplication-errors in division that had a frequency of one per test paper	31	59	46	51	66	53	67	42	59	62	47	583
9.	Number from Group 8 that were found in the list of errors on multiplication test	0	0	0	0	0	1	2	3	0	1	1	8
10.	Number of papers, exclusive of Type 1, that contained no errors in multiplication test	21	37	24	29	34	35	27	26	14	21	15	283
11.	Number of papers, exclusive of Type 1, that contained no multiplication-errors in division	3	6	4	15	15	10	3	13	26	23	13	131
12.	Total number of errors in multiplication test	9	27	13	29	38	19	52	46	45	51	42	371
13.	Total number of multiplication errors that were found in division	40	81	65	67	83	66	107	52	75	72	56	764
14.	Total number of multiplication-errors common to both multiplication and division tests	0	0	1	3	0	2	6	5	2	2	1	22
15.	Total number of students tested	55	95	87	103	94	92	88	106	124	131	95	1075

with multiplication in division, the remedial work to be applied should not consist of multiplication *per se*. The pupil should be given division examples with emphasis placed on the multiplication process. The multiplication involved in division is a specialized ability and not a generalized reaction to a multiplication fact.

The results of the responses to the significant facts show that a reliable diagnosis could not be made concerning a pupil's knowledge of the facts under the conditions described in this study. If it is true that lapses vary during different time intervals, the reliability of the diagnosis will be dependent upon the particular "mind set" of the individual at the time the responses to the facts are made.

The results from the test in the multiplication facts and from the test in division which used the former facts soon pointed to the conclusion that the zero facts in both multiplication and division should be treated separately from the other significant facts. There were four pairs of zero facts on the multiplication test. They were: 2 x 0; 4 x 0; 5 x 0; and 6 x 0. Thus, there was a possibility of eight errors in which zero was a factor of the product. In the division test, zero was found in three different positions in the quotient. They were: (1) final in the quotient only; (2) final in both dividend and quotient; and (3) not final in the quotient only. There were three examples in which zero was final in the quotient only.

In the test already cited,[7] the examples were numbered *h*, *i*, and *l* in which zero was final in the quotient only. Example *O* contained zero final in both dividend and quotient and Example *m* contained zero not final in the quotient only. Therefore, it was possible for a zero error in the quotient to occur five times on the test.

The results of the responses to the significant facts show that a reliable diagnosis could not be made concerning a pupil's knowledge of the facts under the conditions described in this study. This situation is not true for the zero facts as can be determined from Table II.

Table I showed that most of the incorrect responses to a multiplication fact were sporadic and due to chance. However, this same situation is not true for the zero facts as can be seen from Table II. Items 1 and 2 in the latter table show that if the incorrect product was given to one of the eight multiplication facts involving zero, 111 out of 115 cases all eight examples were incorrect. When zero was multiplied by a number, the incorrect answer always given was the number itself, as

[7]Grossnickle, Foster E. *Ibid.*

TABLE II

RELATION BETWEEN MULTIPLICATION-ERRORS FOR THE ZERO FACTS IN MULTIPLICATION AND MULTIPLICATION-ERRORS FOR THE SAME FACTS IN DIVISION FOR GRADES V-XV

	ITEMS	Grade											Total for All Grades
		V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	
1.	Number of zero-errors in multiplication that occurred eight times on each test paper	2	4	4	6	20	13	17	6	24	12	3	111
2.	Number of zero-errors in multiplication that occurred fewer than eight times on each test paper	0	0	0	1	1	0	0	0	0	1	1	4
3.	Number of zero-omissions in the quotient that occurred three times on each test paper	9	13	4	19	21	16	14	9	4	5	5	113
4.	Number of zero-omissions in the quotient that occured four times on each test paper	2	7	1[a]	4	9	6	8	2	1	1	1	42
5.	Number of zero-omissions in the quotient that occurred five times on each test paper	0	2	1	3	6	1	2	0	0	1	0	16
6.	Number of zero-omissions that occurred less than three times on each test paper.	1	0	0	1	0	2	1	1	0	0	1	7
7.	Number of zero types of errors that occurred in both tests of multiplication and division	0	0	0	1	3	1	2	0	2	2	1	12
8.	Total number of students tested	55	95	87	103	91	92	85	103	124	131	95	1075

$2 \times 0 = 2$. Since the zero-errors were consistent in over 96 percent of the cases, one multiplication fact involving zero constitutes a valid basis on which reliable diagnosis may be made concerning a pupil's knowledge of the zero facts in multiplication. It must be remembered that zero may be multiplied by a number, but it is never necessary to multiply a number by zero.[8]

Items 3, 4, 5, and 6 in Table II pertained to the zero difficulties encountered in long-division when the divisor was a one-figure number. In each correct quotient involving zero, the latter symbol was omitted. Items 3 and 6 are significant because they show the consistency with which a zero-error was made on the three examples that contained a final zero in the quotient only. In 113 out of 120 cases, or over 94 percent of the total, if one of the final zeros in the quotient was omitted, the other two likewise were omitted. Item 4 shows the number of cases in which zero in the quotient was omitted when zero was final in both dividend and quotient. Item 5 represents the number of omissions in which zero was not final in the quotient only. From these data it is readily seen that when zero was final in the quotient only as in the example $2\overline{)461}$, the most difficult zero type possible in division was encountered. Furthermore, if a pupil could solve an example in which zero was final in the quotient only, he was able to deal with all other cases in which zero occurred in the quotient. There was not one case of zero omissions from the quotient in which zero was final in both dividend and quotient, or not final in the quotient only, that did not have omissions in the type where zero was final in the quotient only. Items 4 and 5 show that the converse of this statement was not true. Therefore, for purposes of reliable diagnosis, one example in which zero is final in the quotient only is sufficient to determine if a pupil can deal with zero in long-division.

Item 7 is significant because it shows the degree to which zero facts in multiplication were common to division. There were 115 papers in which the zero facts were missed in multiplication. In division there were 120 papers in which zero was omitted from the quotient. From these two groups of zero errors, there were only 12 which were common to both. Hence, ability to use zero in multiplication is not indicative of ability to use it in division or vice versa.

SUMMARY

This study was made to see if ability to give the correct products to the multiplication facts would transfer to the use of these facts in long

[8] Grossnickle, Foster E. "Still More Ado about Zero," *Elementary School Journal*, XXXIII (January, 1935), pp. 358-64.

division when the divisor is a one-figure number. A test containing 50 multiplication facts was administered to 1075 pupils in Grades V-XV, inclusive. Then a test in long division was given which used these same multiplication facts. All incorrect products given in both tests were analyzed to see if an error occurring on one test would be found on the other. The following results were obtained:

1. There were over twice as many incorrect products given to the significant facts in the division test as were given to the same facts in the multiplication test.
2. From the total incorrect products to the significant facts given on both tests, only about two percent were common to both tests.
3. A pupil's knowledge of a multiplication fact was not constant at all times.
4. For diagnostic purposes, one multiplication fact involving zero gave a reliable basis to determine if the zero facts were known in this process.
5. For diagnostic purposes, one division fact on which zero was final in the quotient only was sufficient to give a reliable basis to determine if the zero facts were known in division.
6. If a pupil did not know the zero facts in multiplication, this was not indicative that he did not know how to use zero in division and vice versa.
7. There is only partial ability to transfer a knowledge of the multiplication facts to their usage in the long-division process.

CONCLUSION

This study showed that most multiplication errors of significant combinations in the division process were sporadic and due to chance. However, the errors to the facts in both multiplication and division were constant in about 95 percent of the cases. Furthermore, these latter errors were distributed throughout all of the grades sampled. Since the zero types were almost constant, these facts should be taught at a grade or mental level at which a pupil can profit by generalizations. When the pupil learns that $2 \times 0 = 0$, he should be able to profit from a generalization about zero facts in multiplication. Likewise, when the pupil learns that the quotient for $461 \div 2$ is 230 and 1 remaining, he should have attained a mental level to profit from the generalization about the function of zero as a place-holder in the quotient.

It was seen that there were very few errors of combinations in multiplication that were found in multiplication in division. Moreover, there were over twice as many errors of multiplication in division as there were when the multiplication facts were isolated. The implica-

tion is that division is an integrated process composed of many particular abilities. Then, when a pupil needs practice on the multiplication facts in division, this practice should be given in division and not in multiplication *per se*. In order to develop the ability to use the basic facts in long-division, more emphasis should be given to synthesis of the component factors involved and less to analysis of skills isolated from the unified process.

THE LEARNING OF DECIMALS IN AN ARITHMETIC ACTIVITY PROGRAM

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IN an earlier paper we reported the results of the first year of our experiment[1] in which we wished to ascertain the degree to which the fundamentals were learned in an arithmetic activity program. The first study was limited to common fractions. Our present investigation involving the same group of children is concerned with decimals which subject is commonly learned in Grade VI.

THE CLASS AND ITS RESOURCES

The experimental class was an ordinary group of 39 pupils in a typical metropolitan school in Cleveland. The classroom had practically no permanent learning equipment and supplies except movable desks. Materials required by the units were improvised and assembled when they became necessary. The intelligence quotient of the pupils on the National Intelligence Tests ranged from 92 to 135, the average being 112.5. The fact that this class was above average did not account for our results. When we divided the class into an upper half, having an average intelligence quotient of 121, and a lower half, having an average intelligence quotient of 104, we found that the two halves accomplished exactly the same degree of mastery on the twenty-seven processes. We do have to admit, however, that the class enjoyed experienced, superior teaching.

THE FUNCTIONAL ARITHMETIC COURSE

The activities which made up the sixth grade course were selected because they were rich in the application of decimals.[2] Each unit was

[1] Harap, H., and Mapes, C. E. "The Learning of Fundamentals in an Arithmetic Activity Program," *Elementary School Journal*, XXXIV (March, 1934), pp. 515-525. This paper gives a more complete account of the procedure and treatment of data than the present paper.

[2] The complete course giving the procedure and the arithmetical computations may be secured in two mimeographed bulletins from the Curriculum Laboratory, Western Reserve University, Cleveland, Ohio. The first bulletin is entitled "Making Household Preparations" and costs fifteen cents. The second bulletin is entitled "Eight Activity Units in Decimals" and costs twenty cents.

based on a meaningful situation in the child's experience in school and in daily life. The experiences were real and varied, including keeping accounts, making talks to classes, comparing evaporated with whole milk, making graphs, selecting a recipe, making cocoa, sampling tooth powders, calculating relative costs, selling glaced apples, figuring profits, and so on. In order to provide a maximum amount of pupil participation the class was divided into about six production groups whenever the product was prepared in bulk. While most of the units were anticipated by the teacher, they were taken up willingly by the pupils because they were related to situations which were meaningful and natural to the pupils. The children were given an opportunity to communicate and associate freely as the need arose. They were given abundant opportunities for self-expression and critical judgment. Every unit had its logical and natural close. Much of the time ordinarily devoted to drill and problem solving was devoted to construction, manipulation, and other activities. Actually, therefore, much less than the customary allotment of time was used in computations.

The thirteen units which comprised the work of the year included the following: (1) School Banking, (2) Keeping Spelling Records, (3) The Community Fund, (4) Using Milk, (5) Making Tooth Powder, (6) The School Fund, (7) Making Furniture Polish, (8) Making Ink, (9) Making Hand Lotion, (10) Making Paste, (11) Making Glacéd Apples, (12) Making Presents for Mother, and (13) Making a Garden. Most of these units were never taught before. Four of the units were based on school activities and records. Five units involved the production of useful commodities commonly bought in stores. These are of an experimental nature in the field of consumer education as well as in the field of arithmetic. The pupils acquired much practical and technical knowledge and skill.

New arithmetical steps did not occur in the order of difficulty. Each new difficulty was taken up when it arose, the explanation lasting a few minutes. No practice sheets nor any other supplementary exercises were used. No tests were given other than the initial and final tests. Thus, the repetition of the twenty-seven processes was limited exclusively to their natural occurrence in the units of work. All the work of the pupils was kept in a notebook, the latter being a complete and exact record of all the computations of the pupils. No textbook nor any other printed material was used in the experimental course.

To illustrate, we give here a brief outline of the unit entitled, *Making Furniture Polish*. As a part of the work relief program the school building was being painted. Stimulated by this, the class decided to make

furniture polish for use in the classroom and in the home. The recipe for one quart was brought in by the teacher. Thirty-seven pupils wished to take four ounce samples which would require 148 ounces or 9.25 pints. Since the recipe yielded 32.75 ounces, it was necessary to increase the recipe 4.51 times. The ingredients included: denatured alcohol, boiled linseed oil, raw linseed oil, benzine, turpentine, and cider vinegar. The exact amount of each ingredient was determined and converted into pints and quarts, since they would have to be bought in these measures. The cost of each ingredient was computed and when the total cost was added, it amounted to $1.05. To make four ounces, it would cost $0.28.

The class was divided into six groups, each group having a chairman and six members. Each group worked at a table on which there were two measuring cups, a large jar in which the ingredients were mixed, four bottles containing boiled linseed oil, raw linseed oil, turpentine, and cider vinegar. The benzine and alcohol were dispensed from large containers by the teacher as each group was ready for them. One pupil read the recipe from his notebook; two pupils measured; two pupils checked the measurements; and the remaining two pupils mixed the ingredients. After the contents were thoroughly mixed, each group tried the polish on some piece of furniture in the room. Finally each pupil filled a four ounce bottle for himself. The price of the classroom product was compared with that of six common commercial preparations. It was found that with the exception of one chain store product none was less than three times as costly.

RECORDING THE DATA

The basic list of twenty-seven steps in decimals which we used for our records was based on the analysis of the Cleveland curriculum center for arithmetic. With the aid of this we kept a daily record of the order in which each step occurred and the number of times each step was repeated. Thus we knew the exact sequence and the number of repetitions of each step for every unit and for the course as a whole.

The list of fundamental steps formed the basis of the initial and final tests containing twenty-seven items of three exercises each. The two forms actually used were designed by the arithmetic curriculum center and were widely used in the city schools. This test was also given to a control group for purposes of comparison. We also repeated the tests on common fractions given a year ago in order to discover the permanent effects of our experiment. The criterion of mastery was the same as that used in the local schools and consisted of at least two correct answers on the three exercises involving a given step. In

addition to the test on the basic steps, the pupils were given an initial and final test in problems applying the basic steps. As a further check on our results the experimental group was given a test on mixed fundamentals consisting of 72 exercises covering common and decimal fractions. This was an excellent appraisal of two years of experimentation. The last two tests were more rigorous tests of mastery since correct answers only entered into the final score. The units were recorded in detail, the actual development of the learning experiences and the arithmetical computations as they arose being shown.

THE DATA

Were the basic steps in decimals learned? As measured by the final test the pupils attained a mastery of 96 percent of the twenty-seven basic processes. Twenty pupils mastered all the steps; nine pupils mastered 26 steps; four pupils mastered 25 steps; and four pupils mastered 24 steps. No pupil mastered less than twenty steps. The data are shown in Table I. The supplementary test of problems applying processes confirmed the results of the basic test, the pupils making a composite score of 94 percent of correct answers. Since many of the basic steps were known by the pupils at the outset of the course we were interested to determine what gain the pupils had made over their initial attainment. In addition of decimals, the pupils increased their initial mastery of 54 percent to 99 percent at the end of the year; in subtraction of decimals the pupils attained a final mastery of 98 percent over an initial knowledge of 43 percent; in multiplication of decimals the final mastery was 98 percent over an initial knowledge of 35 percent. The most complete and conclusive gain is shown in the case of division of decimals where the initial knowledge was one percent as compared with a final mastery of 93 percent. The test on mixed fundamentals, consisting of seventy-two items completely covering fractions and decimals bore out the evidence already presented. Out of a possible average of 72 correct answers, the class attained a score of 64.7 or an average of 90 percent accuracy.

How did the experimental group compare with the control group?— For purposes of comparison, the initial and final tests on fundamentals were given to a conventional class taught by an experienced teacher with a good record. Although we had test data for 31 pupils in the control group we could match only twelve pairs because of lack of intelligence records and because of lack of equivalent pairs. The data in the following table show that the experimental group mastered an average of

TABLE I

DISTRIBUTION OF PUPILS ACCORDING TO NUMBER OF ARITHMETICAL STEPS MASTERED IN ARITHMETIC ACTIVITY CLASS IN GRADE VI

Number of Steps Mastered	Percentage of Steps Mastered	Pupils Mastering Steps	
		Number	Percent
27 (all)	100%	20	51.2
26	96	9	23.0
25	92.5	4	10.3
24	88.8	4	10.3
23	85	1	2.6
20	74	1	2.6
Total		39	100.0

26.2 processes or 97 percent of the total, as compared with a mastery of 18.2 processes or 67 percent of the total for the control group. We concluded that the experimental group did better than the control group.

TABLE II

COMPARISON OF EXPERIMENTAL AND CONTROL GROUP

	Pupils		Number of Steps Mastered	
	Experimental	Control	Experimental	Control
1.	Wa 121	Os 121	27	25
2.	Gr 113	Fo 113	24	25
3.	Po 110	Cl 110	27	27
4.	Ac 118	Ho 118	26	24
5.	Pr 100	Ri 101	24	17
6.	Ho 99	Fi 100	26	12
7.	Bo 98	Ke 99	27	17
8.	Da 95	Pa 93	27	14
9.	Ba 94	Jo 92	27	10
10.	St 92	Mi 91	26	22
11.	Sc 92	Sw 91	26	23
12.	Bo 92	Fi 91	27	22
	Average number of steps mastered		26.2	18.2
	Percent of steps mastered		97	67

What evidence is there of the lasting value of the experimental program? We are able to report here the results of the same test on the fundamentals of fractions given three months later, six months later, and one year after the completion of our first experiment. The following table shows a slight change over the summer vacation followed by progressive gains and culminating in a final achievement of about 97 percent. This is the result of cumulative incidental learning, although the work of the year was not devoted to the subject of fractions.

TABLE III

RESULTS OF RETESTING THE EXPERIMENTAL GROUP ON THE FUNDAMENTALS OF FRACTIONS AT LATER INTERVALS

Number of Pupils	Time of Test	Percent Mastery
Multiplication		
37	June, 1933	79.0
26	September, 1933	82.7
32	January, 1934	84.7
35	June, 1934	97.8
Division		
37	June, 1933	83.0
27	September, 1933	78.6
30	January, 1934	86.1
35	June, 1934	96.1

TABLE IV

ORDER IN WHICH STEPS OCCURRED IN THIRTEEN ARITHMETIC ACTIVITY UNITS IN A SIXTH-GRADE CLASS

Steps Included	Order of Occurrence of Steps												
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII
1. Add dec. frac. to hundredths, Sum equal to less than a unit	1							14	9	7	1	10	
2. Add dec. fractions to hund. Sum an integer or dec. mixed number	11	13					7			12	6		
3. Add integers, dec. frac. and dec. mixed no. to hundredths	2	14		5	1	1	1	8				1	
4. Add integers, dec. frac. and dec. mixed no. to thousandths			7									14	
5. Add integers, dec. frac. and dec. mixed no. to ten thousandths			6				6		6				
6. Sub. dec. frac. and dec. mixed number to hundredths no empty spaces in dec. frac.	13	7	9			3				13			
7. Sub. dec. frac. expressed in hundredths zero in tenth's place, in difference	10	8	13				12				4		
8. Sub. integer from dec. mixed no. dec. frac. expressed in tenths or hundredths			11			4							
9. Sub. a dec. frac. or dec. mixed number from an integer; dec. frac. expressed in tenths or hundredths	14	2	10			2		9					
10. Sub. dec. frac. or dec. mixed no. to thousandths, dec. frac. in minuend, expressed with the same or fewer numbers of places than dec. frac. in subtrahend	9	4	8	7			10				7		
11. Sub. dec. frac. or dec. mixed no. to ten-thousandths	15		12										
12. Mult. tenths, hundredths and thous. by integer and vice-versa		6	16	2				6	4	2	5	3	5
13. Mult. dec. mixed no. by integer and vice-versa		5	2	1			5	3	10	8	2	2	10
14. Mult. dec. frac. or dec. mixed no. dec. points appearing in both multiplicand and multiplier product not requiring the prefixing of zeros			15	4			4	2	2	5		5	6
15. Mult. dec. frac. by dec. frac. Product requiring the prefixing of zeros			3				8	11	1	3			
16. Mult. using aliquot parts of a dollar	11												
17. Mult. any no. by 10,000, 1000 by moving the dec. point			1									13	
18. Div. a dec. frac. or mixed no. by an integer. U. S. money	8				3			5	11	11		8	2
19. Div. a dec. frac. by integer, zeros follow the dec. point in quotient	7							12	12	6	3	12	7
20. Div. dec. frac. or a dec. mixed no. by integer when it is necessary to annex one or more zeros to make quo. come out even	6		17					1	3	4			
21. Div. a dec. frac. or a dec. mixed no. by an integer when quo. ends in circulating dec.	5	12	5		3		2	10	8	9		6	4
22. Div. dec. or dec. mixed no. by a dec. frac.		10	14				11	15					
23. Div. a dec. frac. or a dec. mixed no. by a dec. mixed number		9	4		6				5		8	9	9
24. Div. an integer by a dec. frac. or a dec. mixed number		3	19				3	7				4	8
25. Changing a com. frac. to a dec. frac.	3						9	4	7	1		11	1
26. Div. an integer by a larger integer	4	1	18		2			13		10		7	3
27. Div. an integer, a dec. frac. or a mixed no. by 10,100 or 1000 by moving the dec. point	12												

Were the steps learned in sequence? It is obvious from an inspection of Table IV that the processes were not learned in order of difficulty. In no unit was there any approach to an orderly learning sequence of the steps in decimal fractions. Yet this did not seem to affect the mastery of these steps.

Was there any relation between the number of repetitions of a step and the degree of mastery of the step? We wished to know whether those steps which occurred only a few times were well learned. If not, we should have been obliged to introduce supplementary drills. We found that the number of repetitions had nothing to do with the degree of mastery of a step. Table V shows this quite plainly. Step 26 occurred 116 times in the work of the year but the degree of mastery did not exceed that of six steps which occurred less than seven times. Step 16 which occurred only once in the entire year was mastered as well as step 26 which occurred 116 times.

TABLE V

DECIMAL FRACTIONS

RELATION BETWEEN NUMBER OF REPETITIONS AND MASTERY OF STEPS

Step Number	Number of Repetitions	Percentage of Mastery	Step Number	Number of Repetitions	Percentage of Mastery
26	116	94.7	19	25	94.7
25	71	100	23	23	82
14	68	100	7	20	97
12	65	100	24	20	76.8
21	61	97	9	18	97
3	61	97	11	18	97
1	41	100	22	17	97
15	40	100	4	6	97
10	36	97	5	4	100
6	32	100	8	4	97
20	32	97	27	3	94.7
18	31	100	17	2	94.7
2	30	100	16	1	94.7
13	25	100			

SUMMARY

1. In our program of thirteen units based on real situations incorporating the fundamentals of decimals, the pupils attained a mastery of 96 percent of the twenty-seven basic processes.

2. The learning of processes in the order of their appearance in the units did not hinder the mastery of the processes.

3. The processes which occurred only a few times were learned as well as those which occurred many times.

4. The experimental group achieved a mastery of 96 percent as compared with a mastery of 67 percent for the control group.

5. Retesting the group in fractions after an interval of a year yielded a mastery of 97 percent over an attainment of about 81 percent a year ago.

6. On a test of mixed fundamentals covering fractions and decimals the experimental group achieved 90 percent accuracy.

7. Finally, our data seem to justify further experimentation of learning arithmetic in socially real situations or units.

THE VALUES, LIMITATIONS, AND APPLICATIONS OF THE FINDINGS OF THE COMMITTEE OF SEVEN

CARLETON WASHBURNE
*Chairman of the Committee**

RECENTLY the chairman of the Committee of Seven received a letter from one of the editors of the *Journal of Educational Research*, beginning: "City after city is making radical shifts in the curriculum in arithmetic, some of them wisely, some of them haphazardly. I think your Committee of Seven material is a decisive factor in the situation. I think it would be a fine thing for all of us if you could write a summary article, giving the work you have done, and stressing the actual application of your findings. Could you present illustrations of how curricula are being modified and showing how your data apply? Can one justify a curriculum in arithmetic that is definitely stepped up to higher grade levels than now? . . . A statement by you in the form of an article discussing detailed, concrete applications of your findings would be invaluable."

That letter clearly indicates the intent and scope of this article.

The Northern Illinois Conference on Supervision (formerly the Superintendents and Principals Association of Northern Illinois) had many years ago a committee on curriculum. It was called the Committee of Seven. By 1921 this committee had practically ceased to exist, and a new Committee of Seven was organized by its present chairman for coöperative research in arithmetic.

Summary of the Methods Used by the Committee of Seven.—First of all the Committee of Seven determined, with the coöperation of 98 superintendents of schools, the grade in which each arithmetic topic was most commonly taught. Teaching material was then prepared for each topic. It was to be used at its most common grade level and at levels one or two grades lower and one or two grades higher. School systems were found which were willing to teach certain of these topics in these several grades, either by advancing the topics in their curricula or by postponing when

*The Committee of Seven consists of Orville T. Bright, Sup't of Schools, Flossmoor, Illinois; Turner Chandler, Principal Burnside School, Chicago; Harry O. Gillet, Principal University of Chicago Elementary School; J. R. Harper, Sup't of Schools, Wilmette, Illinois; Raymond Osborne, Principal Francis W. Parker School, Chicago (temporarily in absentia); O. E. Peterson, Head of Department of Education, State Teachers College, DeKalb, Illinois, (Secretary); Howard Storm, Sup't of Schools, Batavia, Illinois; and Carleton Washburne, Sup't. of Schools, Winnetka, Illinois, Chairman.

necessary, although there was enough diversity in grade placement to enable the Committee to find schools which habitually taught the subject in a grade lower or higher than that which was most common. The willingness of school officers and teachers to coöperate was one of the most gratifying features of the Committee of Seven experimentation. It showed a genuine desire on the part of superintendents and teachers to analyze their arithmetic curricula in terms of research.

All the children in the grades which were going to participate in an experiment were given group intelligence tests and tests in the foundations thought necessary for the learning of the process. Foundations tests always included the prerequisite knowledges or skills, such as, in the case of compound multiplication, knowing the multiplication facts, ability to multiply numbers of two or more digits by numbers of one digit, and ability to add numbers of several digits. Sometimes the foundations tests also involved an understanding of the concepts back of the process which was to be learned and an understanding of the number values, such as the nature and values of fractions. Weaknesses revealed in these foundations tests were overcome, as far as possible, by direct teaching before the experiment was begun, and a second form of the foundations test was then given. It was on the basis of the scores on the second form of the foundations test that graphs were later made.

Tests were prepared to cover the more important elements of the teaching material. These tests were made in several forms to include the same elements of difficulty in the same order. One of these forms was used as a pre-test, others were used as teaching tests, one was used as a final test given at the close of the teaching, and one as a retention given six weeks later with no intervening teaching of the topic.

The methods of teaching, the time allotted for each day, the number of weeks the teaching was to continue, and so on, were all specified by the Committee on the basis of preliminary experiments in their own schools. No home work was allowed.

All tests, including intelligence tests and foundations tests, were sent to the Research Department of the Winnetka Public Schools for tabulating. Mabel Vogel Morphett, Director of the Research Department (and during her absence for several years, Vivian Weedon), and William H. Voas, Psychologist for the Winnetka Public Schools, have worked with the Committee of Seven continuously. Indeed, without the facilities of the Winnetka Research Office, the work of the Committee would have been almost hopelessly handicapped.

The resulting tabulations were studied in a variety of ways, but the most effective and useful form of summary was a graph for each topic, made as follows:

First, the papers were eliminated of all children whose pretests showed that they had already learned the process. Second, regardless of mental age, the achievement on the retention test was plotted against foundations test scores. This usually resulted in a straight line graph, rising steeply from practically no children able to do the process when the foundations test scores were very low to a considerable percentage of children able to do the process when the foundations test scores were high. An inspection of these graphs made it possible to determine the minimal degree of mastery of the foundations which was necessary for learning the topic in question.

Third, the papers of all children who had not reached the necessary grasp of foundations were eliminated and the remaining papers were tabulated against the mental ages of the children. In other words, given the prerequisite knowledge and skill for a given process, at what mental age can a child best learn that process? The resulting graph usually was the typical S-curve of growth. Mental ages were plotted along the base of the graph, the percent of children who showed 80 percent retention being plotted up the side. The curve showed usually that the longer one postponed the teaching of the topic up to a certain point, the more one gained by the postponement. Then came the point of flexion in the curve, and the more one postponed thereafter, the less one gained by the postponement.

The point of flexion in the curve was usually considered the ideal point for teaching the topic, provided at least three-fourths of the children were able to achieve success at this stage of mental growth. In some cases the point of flexion did not come until considerably after three-fourths of the children were able to achieve satisfactory mastery. In these cases a minimum mental age was set up, it being the mental age at which three-fourths of the children could achieve 80 percent mastery of the topic.

Other graphs, freed from the arbitrary standard of 80 percent mastery by three-fourths of the class, were prepared on the basis of the average achievement at each mental age.[1] Interestingly enough the points of flexion on these curves came at practically the same places as did those on the curves of the other graphs. In short, there seems to be a stage in a child's mental growth before which he cannot learn the topic economically and after which he can. That stage of mental growth is the one which the Committee of Seven has recommended for the placement of each arithmetic topic.

[1]Washburne, Carleton. "Grade Placement of Arithmetic Topics," *Twenty-Ninth Yearbook of the National Society for the Study of Education* (1930), pp. 641-670.

In this way, some twenty-two processes and sub-processes have been investigated and reported.[2] The mental age level assigned by the Committee of Seven is, of course, the mental age at which three-fourths of a normal group can attain scores of 80 percent or better in retention tests covering a topic. We do not know how much earlier they can begin to advantage, in learning a topic, nor when the easier aspects of a topic might be learned to advantage. Within the limits of the experimentation, the findings show that to add columns of a certain height and width requires a certain degree of mental development, to do long division with a specified length of divisor and dividend and involving other specified difficulties requires another level of mental ability, and so on. The Committee's recommendations in regard to the placement of these various topics have been repeatedly published (see bibliography).

VALUES OF THE STUDY

The work of the Committee of Seven has been the first thorough study of the curriculum in relation to mental age and necessary foundations.

It has brought out strongly the need for knowing the child's mental maturity and his readiness for learning a given topic. Schools continually give too little attention to both factors, not only in arithmetic but in all other school subjects. They attempt to teach children before the children have reached the necessary mental maturity; they attempt to build on inadequate foundations. The result is waste, inefficiency, and frequent failure.

That failure has been more common in arithmetic than in any other school subject above the first grade is common knowledge. The work of the Committee of Seven has shown one of the principal causes of that failure and has brought out the means of prevention.

The techniques worked out by the Committee in its studies have suggested ways of grade placement research in all fields, at least all fields having to do with the acquisition of skills—not that the techniques of the Committee of Seven are in any way perfect (their limitations will soon be pointed out), but at least they are the first techniques to be developed in this field, are workable, and produce practical results.

The very limitations of the work of the Committee of Seven point the way to future research, and should stimulate such research.

Finally, there are immediate practical results in the tentative norms as to the necessary maturity and readiness for learning each arithmetic topic. These are already affecting practice widely.

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[2] Gillet, Harry O. "Placement of Arithmetic Topics," *Journal of the National Education Association* (June 1931), pp. 199-200.

LIMITATIONS OF THE STUDY

Some of the limitations of the Committee's work are those inherent in any new venture. The early experiments, particularly, showed the lack of pre-existing experience in the field of study. Other limitations are inherent in the fact that the members of the Committee are all very busy people, each with a full time job. They have to do their Committee work on Saturdays, evenings, or an occasional school day. Fortunately, in the experimentation this was largely compensated for by the fact that the director of the Winnetka Research Office was able to devote practically full time to the Committee work, and the various clerical assistants were likewise on full time whenever necessary. Then there were and are financial limitations. The work of the Committee has been financed almost entirely by contributions ranging from one to two hundred dollars each from the parent organization and the northern divisions of the Illinois State Teachers Association. Before the depression three grants of $1,000 each were made by private individuals. The money has not been sufficient to do the repeating of experiments and to do the minute detailed analyses that the Committee would like to have done.

One serious limitation of the Committee's experiments lies in the fact that it necessarily had to dictate for each topic a specific method of teaching and specific materials. While this method is probably equal if not superior to the method used in most schools, it undoubtedly could be improved upon. Whether improvement in the method would change the mental maturity placement is, of course, purely hypothetical; i.e., we do not know whether among good methods of instruction one will require a maturity level for completing the learning of the process significantly different from that required by another. It is quite possible that a truly functional method growing out of a felt need on the part of the child, tied up closely with the realities of his life, would make it possible for him to learn a process at a period earlier than that indicated by the Committee's research. On the other hand, it is perhaps equally possible that if one were to use a truly functional method, one would have to postpone the teaching still farther, in order to have the process function in the child's life. We are in the field of mere speculation when it comes to the question as to how far change of method would change the apparent maturity necessary for learning a given topic. Research in this field is highly desirable.

Again, we do not know what the effect would be of changing the time allotment for teaching. The Committee specified the amount of time necessary, as shown by informal preliminary experiments, for covering the ground of the process rather thoroughly. This, however, was for con-

centrated instruction in the one process. It is possible that a long, gradual approach might either postpone or advance the point at which a child could learn the process. The fact that on the whole there is relatively little loss between the final test and the retention test, would seem to indicate that the instruction was not so concentrated as to be a kind of cramming. It would, however, be interesting to experiment with the effect of a slow spiral attack upon arithmetic topics, in comparison with the more direct attack used by the Committee of Seven.

The standard of 80 percent retention by three-fourths of the class, as indicating the minimum of acceptable mastery, was of course purely arbitrary. The Committee has graphed the results of using various standards, and whereas, of course, if the standard is raised, the number of children at any given mental level reaching mastery will be lowered, and vice versa, it is interesting to note that the graphs are all of the same shape, i.e., the curve breaks at the same mental age level. As already pointed out, the method of simply taking average scores of each mental age level, regardless of any arbitrary standards, tends to show that the breaking point of the curve is independent of arbitrary standards. Nevertheless the *minima* set by the Committee of Seven in their published tables always assumed the arbitrary standard of 80 percent retention by three-fourths of the class, and anyone who is willing to have lower retention or a smaller proportion of the class achieving that retention, could perfectly freely drop below the minimum standard set by the Committee. The converse is likewise true.

The foundations tests as prepared by the Committee are inadequate. They are much better than most schools use, but, except for the foundations for fractions and for the early work in addition, the foundations tests have lacked elements that would show whether or not the child had a real concept of the meaning of the process that he was going to undertake. The Committee's foundations tests, for the most part, check only his mechanical facility in the prerequisite skills.

The tests in the process taught (pre-tests, final tests, and retention tests) have not always been sufficiently complete. Sometimes one level of difficulty in the process was represented by only one example. This makes it unsafe to generalize on the placement of each type of subprocess included within a given process. For example, there may be a very real difference in the proper grade placement of adding such unlike fractions as $\frac{1}{2}$ plus $\frac{1}{4}$, and adding such unlike fractions as $\frac{3}{8}$ plus $\frac{1}{6}$. The Committee's test on addition of fractions does not sufficiently differentiate between these two degrees of difficulty in the addition of unlike fractions. The same is true for tests in some other topics. Some

of the tests were sufficiently complete to make such analyses possible, but lack of time and money have prevented completion of the studies.

SUPPOSED LIMITATIONS

Besides these real limitations there are difficulties that arise in the minds of some critics. These should be discussed briefly.

1. The coefficient of correlation between mental age and achievement is on its face much too low to indicate a definite predictive value for the individual child as to when he can successfully attempt the subject. I think the answer to this criticism lies in the fact that there is evidence that the percentage of children who can achieve mounts rapidly to a certain point of mental age, and that it mounts very little thereafter. As a practical piece of school procedure this is sufficient evidence that postponing to this point will materially increase the number of children who are successful, and that postponing beyond this point will yield diminishing returns in terms of the proportion of children who will be successful in attacking the subject. The fact that there is too much scatter of scores at each mental level to produce a high coefficient of correlation in no way affects the fact that the postponement up to a certain level will greatly increase the probability of the child's mastering the topic.

2. Occasionally in the table of norms published by the Committee of Seven the minimum standard for the mastery of a prerequisite topic is higher than that for mastering the topic which is based upon it. For example, the minimum mental age for short division is given as 11 years 4 months, while the minimum mental age for long division is given as 10 years 9 months. Yet short division was considered one of the prerequisites for long division and is included in the foundations test for long division. This looks like a serious discrepancy, but as a matter of fact it is not at all. It must be remembered that the mental ages given are only for those children who have *passed the necessary foundations test*, i.e., all children with low foundation scores are eliminated before plotting the proportion of children who are successful at each successive mental age. It is therefore possible and true that three-fourths of the children do not achieve 80 percent mastery of short division until a mental age of 11 years 4 months. At the same time it is equally possible and true that of the children who have achieved this mastery of short division (and of course there is always a considerable percentage who achieve it before the mental age of 11 years 4 months), three-fourths can achieve a satisfactory mastery of long division at a mental age of 10 years 9 months. This does not mean that three-fourths

of the children of 10 years 9 months can learn long division. It means three-fourths of the children who at 10 years 9 months have mastered short division—and this would not be the majority of all the children of this mental age. The optimum mental age for teaching long division is published as 12 years 7 months, whereas the optimum for short division is 12 years 2 months. In other words, the break in the curve does occur, as one would expect it to, at a higher mental level for long division than for short division, and it is desirable to postpone long division until this break in the curve occurs.

3. Some critics have spoken of the danger of accepting the results at face value because of the necessary tentativeness of the Committee's findings. Is it not always better to act on the basis of even tentative scientific results than on the basis of a tradition which has always been unsatisfactory and which has resulted in many school failures?

4. Will not the following of the Committee of Seven recommendations result in a crowding of the upper grades? As a matter of fact, it doesn't. The upper grades have been crowded in the past with long and arduous reviews of half-learned material. In the Winnetka schools, for example, our upper grades have actually had more time clear for arithmetic since the adoption of the Committee of Seven curriculum than they had before. The children go at a topic, which has been postponed until they are ready for it, with a zest and an efficiency that far exceed, if the teacher's reports are to be believed, the way they went at a review of old topics half-learned.

A CURRICULUM BASED ON THE COMMITTEE OF SEVEN FINDINGS

Some textbooks have recently been revised, partly, I understand, at the behest of the State Textbooks Commissions of New Mexico, California, and Montana, in an attempt to meet the Committee's recommendations. These attempted revisions, while a step in the right direction, fail to take cognizance of the fact that it is not the grade in the usual school sense that is important, but the mental age and the foundations of each child. Unless adequate foundations tests are provided for the beginning of each new process, and unless the teacher is instructed as to the importance of knowing whether the child who is going to undertake the process is ready mentally and in terms of foundations, the mere postponing of a topic one or more grades will only partially solve the problem of teaching each arithmetic topic at the most desirable level.

The Committee does not feel that its investigations are sufficiently extensive to justify its recommending a complete arithmetic curriculum. Measurements and denominate numbers, for example, are still being in-

vestigated. The approach to a topic is terra incognita. Should it be very gradual and based first on informal experiences, or should it be a direct attack, which for experimental purposes was almost essential in the Committee's work? Is the level at which the Committee finds that a topic can most successfully be taught to reasonable completion necessarily the level at which the instruction should first begin? We frankly don't know. Is the fact that a child *can* learn the 64 easy addition facts in the first grade (i.e., at a mental level of 6½ years 5 months) evidence that he *should* do so? Would he perhaps be better off postponing such formal work until he has gained more experience? On this we have no evidence in the Committee's findings.

And then there is the fact that arithmetic is much more than mere computation—at what stages of a child's growth can he best get the basic arithmetic concepts? When can he best use appropriate arithmetic techniques in getting certain types of social understanding, insights, and appreciations? All of these questions bear vitally on curriculum building, and no research as yet throws light upon them.

One word more of caution: For the sake of convenience, we are using the terms 1st grade, 2nd grade, etc. The grade must be interpreted, however, not as the grade in which the child is sitting, but as the mental level which he has reached. A mental age of 6½ to 7½ is considered first grade, 7½ to 8½ second grade, and so on. A child sitting in a third grade room, but having a mental age of 9½ or more would from the Committee's standpoint be considered a fourth grade child. Similarly, a child sitting in a third grade but having a mental age of 7½ would be considered a second grade child. It is not the room the child is sitting in, but his mental growth as evidenced by intelligence tests, by foundations tests and previous work in arithmetic, and by the teacher's observation of his approach to the subject that must be considered. The Committee wishes to emphasize very strongly the importance of giving a foundations test before each process. It should measure prerequisite skills and basic underlying concepts necessary for the learning of the new process. Adequate foundations are even more necessary than, although of course closely related to, the attainment of a given mental age.

Here, then, are some of the immediate, practical, albeit tentative, applications of the Committee's work. These applications have been made and are continuously being made in the schools of all the members of the Committee of Seven, and of course in many other schools as well. All of us who have used these findings are convinced that such

use tends to reduce materially retardation in arithmetic, and particularly to reduce or eliminate the grief of long, hard, and unnecessary drills. We have all found it practical and eminently desirable to use these findings in our curricula:

The only formal work in arithmetic that can be adequately mastered by first graders is the addition facts with sums of 10 and under. This does not mean that very easy subtraction facts cannot be learned at this level, but that the large group of the easy subtraction facts measured by the Committee of Seven (the easiest 50) are not successfully learned at this level. To avoid repetitiousness, let it be said here, once and for all, that the statement that a topic must not be taught below a given level never means that certain easy aspects of it may not be introduced at a lower level. We do not know whether they may or may not.

In second grade, children are able to learn the remaining addition facts and the easier half of the subtraction facts.* They are also able to learn how to add 1-digit numbers to 2-digit numbers.

Children of third grade ability can learn to add columns 3 digits wide and 3 digits high, can complete the subtraction facts,* and can learn to subtract 3-digit numbers from 3-digit numbers.

Children of fourth grade ability, especially toward the latter part of the fourth grade, can learn all the multiplication facts and can learn to use these in both simple and compound multiplication where the multiplicand does not exceed 4 digits and where the multiplier does not exceed 3 digits. These children can also learn the meaning of fractions* and can learn to make and read simple bar graphs which do not involve computation and fractional estimates.

At the fifth grade level of ability children can add columns 4 digits high and 3 digits wide, can readily learn the division facts and apply these to short division, can learn to add and subtract fractions and mixed numbers with like denominators,* and can learn to add, subtract, and multiply decimals.

Children of sixth grade ability can learn the more difficult division with two place divisors and 4 place dividends,** including division of decimals by integers. They can learn to multiply and divide fractions.

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*Subjects marked * are placed at the optimum level, others at the minimum. This arrangement makes full use of the Committee's findings, but, to get balance in arrangement, uses optimum placement instead of minimum in several cases.

**The Committee made several studies of long division and the minimum mental age for completing different aspects of it ranged from as low as 9 years 10 months for the simplest forms to 12 years 7 months for the complete process involving all difficulties. Sixth grade placement is a trifle low for the complete process, but is actually used in the Committee's schools.

Children of the seventh and eighth grade levels can do Case I percentage and can learn to add and subtract unlike fractions involving the finding of common denominators, to divide decimals by decimals and to do Case II percentage.

If the child does not undertake to complete any of these topics until he has reached the mental age usual for the grade to which the topics are above assigned, and if he has a thorough grasp of the necessary foundations, his chances of successful achievement are very much greater than under the traditional curriculum.

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A progress report of the Committee of Seven for 1935-1936. Contains teaching units and tests in measurement of length and surface.

EDITORIAL

OUR UNINTEGRATED COURSES IN PROFESSIONAL EDUCATION

It is difficult to turn through the announcements of professional courses in most of the larger institutions for the training of teachers without being impressed by their numerousness. This is particularly true of graduate courses in education: courses in the nature and needs of society, child development, curriculum, methods, educational psychology, measurements, history of education, philosophy of education, elementary education, secondary education, etc. While it is probably true in our efforts to secure a certain amount of expertness in the different phases of professional education that we have introduced some needless multiplication of courses, it is not the purpose of this paper to comment upon this phase of the subject but upon the problem of integration raised by these courses: How, with so great a variety of educational courses, is the student of education to secure an integrated conception of the educative process as a whole, such as might be expected of him in his everyday practice of education? Reference has frequently been made to the fact that educationalists often hold at one and the same time conflicting theories of education. Many persons report that the courses in education have left them confused, and without an integrated view of the educative process. While this situation may be due in part to our educational superficiality (and education is not different from other subjects in this respect) it seems also to be due in part to the non-integration of a highly compartmentalized system of education.

There appear to be at least three levels of integration appropriate to this problem: (1) the administrative level; (2) the instructional level; and (3) the learner's level. The ultimate hope of the educationalist is somehow to get the learner integrated. On the administrative level one of the most important problems of integration arises in the classification of course offerings. For many reasons, the groupings of courses in educational announcements (bulletins, catalogues, etc.) have continued chaotic. An elementary principle of classification is that the categories employed in the classification of things be chosen from some consistent point of view. This principle appears to be frequently violated in the grouping of education courses. If there is a department of child study, one would expect to find, for example, also groups of

courses relating to the teacher; the curriculum; the materials of instruction: textbooks, supplies and equipment; and the socio-physical environment for learning. If the groups of courses are made according to the divisions of the school system one would expect to find groups of courses relating to elementary education, higher education, etc. The courses also may be grouped according to function: administration, supervision and instruction and the latter further subdivided into educational curricula, objectives, methods, materials and measurements. Sometimes the courses are divided by positions: superintendents, principals, special supervisors, school nurses, directors of guidance and research and teachers. Frequently the courses are grouped according to subject matter fields: history of education, philosophy of education, educational sociology, educational psychology, methods, measurements and scientific techniques. Seldom does one find any one of these bases employed exclusively in the grouping of courses in an institution, but several or all of them may be employed simultaneously, producing a complete hodge-podge and leading to much needless duplication of content characteristic of so many courses in education.

There are many problems of integration on the instructional level. A major problem of integration on this level grows out of the necessity of constantly relating the content of particular courses to the instructional program as a whole. Many instructors it would appear fail to make this very important connection. Beyond this necessity one finds within the field of education many conflicting theories, principles and patterns. What passes for the truth depends all too frequently upon the field or instructor that gives rise to it. This situation is well illustrated in the field of method. There are at least three groups of methodists in the field of education: first, the special methodists; second, the general methodists; and third, the elite of the methodists, the educational psychologists. The traditions and conceptions of these three groups are all different. Nowhere does the student get as he should an integrated view of the educative process, tying together goals, means, methods, materials and outcomes. The course offerings in the field of learning itself fall into three groups: first, courses discussing theories of learning; second, courses discussing mental-motor processes; and third, courses discussing the principles and conditions for economical learning. These different approaches to learning and to the methods of classroom instruction are seldom brought together in any sequential relationship setting forth the part activities in the total process: (1) objectives, (2) the learning situation (subject matter to be learned or activities to be participated

in), (3) the mental motor processes involved, (4) the principles and conditions for economical learning, (5) the principles of method, (6) techniques of teaching, (7) materials of instruction; and (8) outcomes and their evaluation. The student may spend a considerable amount of time in courses in education and educational psychology without getting anything that approaches an integrated view of teaching. In the absence of such, the student is unable to approach the everyday problem of teaching as effectively as he should. Besides these fragmentary and conflicting points of view in education, illustrated from the field of method, the student of professional education is further confronted with the many special educational biases: the science bias, the history bias, the philosophy bias, the elementary education bias, the secondary education bias, the measurement bias, and many other biases, with all of which he must contend.

The ultimate goal of administrative and instructional integration is the integration of the learner. Instructional and administrative integrations can only pave the way. The actual tying of things together must be done by the learner. The instruction and the administration can furnish the means, emphasize its importance, and note progress; the student must supply interest, effort, and mental activity. The student's best measure of progress toward integration may be found in applying educational theories to concrete teaching, administrative and supervisory situations. In some such way he may be led to bring his many conflicting theories of education into some logically consistent whole.

A. S. Barr

REVIEWS

Address all communications relative to book reviews to Editor Kai Jensen, University of Wisconsin, Madison, Wisconsin.

HOVIOUS, CEROL. *Following Printed Trails* (New York: D. C. Heath and Company, 1936), pp. x + 371.

It is well known that improvement in reading proficiency is marked from the first through the fourth grade and slow thereafter. This striking change in rate of progress at about the fifth grade coincides with important changes in reading instruction. From the fourth grade on there is much less emphasis upon the teaching of reading and more upon content subjects such as arithmetic, grammar, and the like. It has been suggested that more attention to reading at the higher levels would be desirable. Hovious' book, designed for high school students, should therefore, fill a long felt need.

The book, after preliminary orientation on the importance of knowing handicaps and the desirability of removing them, furnishes instruction and materials for improving the various phases of reading such as: finding main ideas, remembering important details, speed, phrasing, meaning of words, eye movements, skimming, etc.

There is little to criticize and much to praise in this book. Exercises for "pacing eye movements" as outlined by the author are neither necessary nor desirable. As reading improves under other forms of instruction, the eyes will automatically adjust themselves since they merely reflect central processes of perception and assimilation. In nearly all instances, however, the materials and exercises in the book are based upon sound practices. The choice and organization of the materials should produce strong motivation and maintain interest at a high level. The exercises are ample. The last two chapters are of special importance to the student. They teach: (1) how to use tables of contents, the index, bibliographies, etc., as reference aids in books; and (2) how to interpret simple tables, graphs and charts.

This book will be gratefully received by teachers and enjoyed by students. It should find a wide use.

MILES A. TINKER
University of Minnesota

GRAY, WILLIAM S., AND LEARY, BERNICE E. *What Makes a Book Readable* (Chicago: The University of Chicago Press, 1935), pp. XVIII + 358.

The authors of *What Makes a Book Readable* have given teachers of adults, librarians, and authors an excellent, single, numerical means of predicting the relative difficulty found in the combined structural elements of reading material for a reader of limited ability. They have made an initial, meticulous attack upon the most measurable phase of the vast problem of readability, and suggest that future studies of content, format, and organization must be made before the problem is solved.

First, they made a survey of current opinion concerning what makes a book readable. One aspect of the findings was isolated for intensive study. It related to the reading ease or difficulty as determined by the presence of measurable, structural elements.

Then one thousand six hundred ninety

representative adults were testsd to discover how well they could read fiction and non-fiction of general interest, found in books, magazines and newspapers. One-sixth of them could read as well as high school graduates; one-sixth could read with the proficiency commonly associated with the lower elementary grades; and the remaining two-thirds ranged between these two extremes. The average reading scores showed that the materials represented varying degrees of difficulty. The significant elements of difficulty were studied, such as the number of different easy and difficult words, of personal pronouns and of prepositional phrases; the percentage of monosyllables, of different words and of simple sentences; and the average length of words and of sentences. A single index of the combined elements was calculated. This is an advancement over former counts of isolated elements, because it is more usable for practical purposes.

Three hundred fifty books suggested by the Hoit list and by forty readers' advisers were rated and the findings are available for use. The authors suggest that there be a central agency for the work of determining the exact degree of difficulty of reading materials.

The average lay reader will be disappointed in this book. In the first place the title will deceive him. He will expect the mystery of readability to be completely solved, not realizing the vastness of the baffling problem. In the second place, the tabular information and the technical passages will be too difficult for him to understand. But the book was written primarily for teachers of adults, and for librarians and authors, who would not accept information as scientific without precise, technical data. It is a most worthy, initial contribution to the serious problem of finding the right reading materials for the reader of limited ability.

MARY WILLCOCKSON
Miami University

JOHNSTON, JOHN B. ***Education for Democracy: Essays and Addresses*** (Minneapolis: The University of Minnesota Press, 1934), pp. VI + 280.

In his own words, "The author seeks to focus attention on the function of the educational system in a democracy.[1] With one exception these papers and addresses have been written since the publication in 1930 of his earlier book on ***The Liberal College in Changing Society.***[2] It is, therefore, possible to compare his former statements concerning the functions of the liberal college with his later views expressed in this collection of occasional papers.

In view of the fact that his chapter of conclusions in the earlier book was written in December, 1929,[3] and since he tells us in the preface that that book was "written literally in odd moments—spared with difficulty from the daily routine of college administration,'"[4] we may safely assume that practically all the book was written before the crash of the stock market in October, 1929. We may, therefore, assume that we have his conception of the function of the liberal college before that event available for comparison with his views as affected by the present crisis.

Dean Johnston opened the first chapter of his earlier book by saying, "The function of the college is to make learning available to society.'"[5] He made his meaning clearer by the further statement that, "The central purpose in making learning available is to enable society to adjust itself with the minimum friction and loss in successive stages of its evolution.'"[6]

With remarkable clarity of vision he called attention to the dangers inherent in the economic situation before the crash.

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[1] p. v.
[2] Johnston, J. B. *The Liberal College in Changing Society.* New York: The Century Company, 1930.
[3] *Ibid.* Note 24, p. 319.
[4] *Ibid.* p. viii
[5] *Ibid.* p. 3.
[6] *Ibid.* p. 38.

"At the present moment," he said, "a new industrial revolution is in progress in the country. Are the colleges aware of it? Are they directing the intellectual forces in their hands to bring about the adjustments which are necessary to prevent suffering, starvation and disarticulation in industry and social life?"[7] Further quotations would only serve to elaborate the point of view so clearly indicated by these few sentences.

In the later book the author states essentially the same conception of the function of the educational system but he has come to see that the completely elective program under guidance proposed in the earlier book is inadequate to meet the needs of society. A few quotations will serve to show the changed emphasis. In the earlier book he says, "The requirements made of each student would be based on a careful consideration of his individual needs, and the whole of his study program would be a means of working out his individuality."[8] "Common prescription of studies for all students is a remnant of the imposition of institutional tradition."[9] "Instead of building a supposedly ideal curriculum for all we should turn our attention to finding out what things among our resources are useful for each of our students.—This does not mean free election by the student nor does it mean prescription by the faculty, but coöperation between student and adviser, *a system of election under advice* in which the faculty bears its share of the responsibility."[10]

Contrast this elective system of the earlier book with the clear-cut demand for universal training in the social sciences and history as stated in the book under review. "In our time," says the author, "the one supreme object of cultural education is the intelligent handling of the problems that arise in politics and mass industry."[11] Students must learn to take an interest in public affairs, to undertake to influence their fellows, and to exercise leadership in politics. For this purpose colleges must give more attention to history and the social sciences, political ethics, and psychology."[12] "In a democracy it is absolutely essential that every voting member be led to exercise and develop to the utmost his thinking and his judgment in all the affairs of individual and community life.[13] "Whatever may be the occupation for which the student prepares himself, he cannot discharge his duties as a citizen and his debt to his fellow men without a reasonable knowledge of the historical development and present working of a society."[14] "The organization of industry, the production of food, the utilization and conservation of natural resources, the framing of laws, the operation of governmental machinery, and the cultivation of customs and ideals conducive to a healthy and satisfying social life are some of the functions for which higher educational institutions have the responsibility to train both expert public servants and large bodies of intelligent citizens. Colleges of liberal arts have failed to realize their duty in this direction."[15] "In addition to training for public service of a specific professional and expert character, the educational system should make a provision for training all other members of society to the extent of their capacity for understanding the products of scientific study and the intelligent application of this knowledge to the life of the whole people."[16]

When the author has dealt so effectively with the social function of higher education, it seems almost captious criticism to point out that his broader purpose as stated in the preface has not been accomplished. The consideration of

[7] *Ibid.* p. 41.
[8] *Ibid.* p. 167.
[9] *Ibid.* p. 42.
[10] *Ibid.* p. 168. Italics as in text.
[11] Johnston, J. B. *Education for Democracy*, p. 75.
[12] *Ibid.* p. 77.
[13] *Ibid.* p. 80.
[14] *Ibid.* p. 247.
[15] *Ibid.* pp. 258-9.
[16] *Ibid.* p. 262.

the social functions of elementary and secondary education is entirely incidental to his main line of thought and is, therefore, inadequate and superficial. The elementary school should provide "a continuous experience in living—. For subjects of study there should be substituted activities suited to the age and abilities of the child."[17] "In connection with these activities will naturally and easily appear all necessary opportunities for learning to read and write the mother tongue, and for the use and understanding of arithmetic."[18]

Thus he gives his stamp of approval to the activities movement in elementary education. But the careful student of education insists that judgment must be suspended until further evidence is at hand. In the meantime many of those most competent in the field of elementary education are pointing out the values which inhere in carefully planned sequences of experience in geography, history, and science.

In a similar cavalier fashion the author proposes a great expansion in vocational training in the secondary school. "Beginning with the junior high school there is no need for the development of additional vocational training courses for the larger part of high school students. Present facilities do not begin to meet the needs of preparing youth for their active lives."[19] Competent opinion on this issue is divided with general education rather than vocational education receiving most support at the secondary level. The most serious criticism of the book is based on the repetition which is inevitable in a collection of occasional papers and addresses. It is to be hoped that the author will find time to revise the book by giving his theme a single, consecutive treatment. In such a revision he might well secure the collaboration of some member of the faculty of the College of Education in his own institution to deal with the functions of the lower schools.

A. K. Loomis

University of Chicago.

[17] *Ibid.* p. 177.
[18] *Ibid.* p. 235.
[19] *Ibid.* p. 144.

RESEARCH NEWS and COMMUNICATIONS

Address all research news and communications to Carter V. Good, Teachers College, University of Cincinnati, Cincinnati, Ohio.

Commission on the Curriculum.—The Commission on the Secondary School Curriculum of the Progressive Education Association, of which V. T. Thayer of the Ethical Culture Schools, New York City, is chairman, is studying a number of important problems: the physical, intellectual, social and emotional development of some 650 adolescents in relation to curriculum reorganization; analysis of the teaching and cultural assumptions in the various subject-matter fields; analysis of common elements of method, and of special methods appropriate to a given field; study of pupil difficulties in various subject-matter areas; etc.

Conference on Business Education.—The Fourth Conference on Business Education will be held at the University of Chicago, June 25 and 26, with the subject of "Business Education for Everybody." The conference of last June drew 205 participants from 21 different states.

India Conference.—It is interesting to note the topics discussed at the Tenth All India Educational Conference in December, 1934: childhood and home education, primary and rural education, adult education, secondary education, vocational education, moral and religious education, health and physical education, training for teachers, internationalism and peace, examinations, and educational experimentation and research. It was recommended that sets of standardized tests, mental and scholastic, suitable for use in India, be prepared.

Harvard Tercentenary.—The Harvard Tercentenary Conference of Arts and Sciences will be held from August 31 to September 12, 1936. Five symposia will be held, dealing with "physical sciences," "biological sciences," "factors determining human behavior," "authority and the individual," and with "independence, convergence and borrowing in institutions, thought, and art."

Peabody Conference.—The Seventh Annual Conference of School Board Members and School Administrative Officers will be held on June 11, 12, and 13, 1936, at George Peabody College for Teachers, Nashville, Tennessee.

Bibliography on Teacher Education.—Pamphlet No. 66 of the U. S. Office of Education covers references on teacher training from June 1, 1932, to October 1, 1935, and supplements Bulletin No. 10, 1933, Volume I, of the National Survey of the Education of Teachers.

1935 Annual Reports of the Carnegie Foundation and of the Carnegie Corporation.—The report of the Carnegie Foundation for the Advancement of Teaching discusses scholarship, the Townsend Plan, mass education, the Pennslyvania Study, the problem of tax exemption, and salary schedules and competition for teachers. W. A. Jessup, president of the Foundation, criticizes the undue emphasis given in some high institutions to trivial publications. "The possession of a doctorate or the multiplication of trivial publications has tended to blind those

who are responsible for selecting, promoting, and making comfortable the teaching staff of our colleges and universities to the fact that personality is still an indispensable element in an institution's effectiveness. Standardizing associations meant well in their pressure on colleges to increase the number of doctors on their staff. This has resulted all too frequently in an accumulation of colorless, superficial scholars who were quick to recognize that the likeliest road to promotion lay in the direction of 'publication.'

"The fact is far too often overlooked that the most fruitful scholars and the finest teachers whom this country has produced, men and women whose lives are reflected in institutional glory and alumni affection, have possessed qualities vastly finer than a flair for 'publication.' "

The program of the Carnegie Corporation in the field of general education has been as follows:

"The Corporation in carrying out its responsibilitiies to education has been steadily moving from its original practice of making gifts to institutions for their general purposes toward a concentration of its efforts upon advancing the cause of general education through research, through support of experimental programs, and in other ways.

"During the year little has been done to start new lines of inquiry of this character, but much has been learned from the progress of enterprises supported in previous years.

"The Pennsylvania Study is still fruitfully engaged upon the digestion and exposition of the unique material it has gathered together, and the interesting plan of the Progressive Education Association to throw light on another angle of the relations between school and college has been launched. The adjustment service maintained in New York City under the general direction of the American Association for Adult Education has proved to be essentially an educational enterprise, and the reports issued by its Director, J. H. Bentley, are educational documents of unusual interest. Another document for which the Association is responsible is a report upon the educational programs of the CCC Camps. Few people realize that this is no longer an experiment quite incidental to the major purpose of the camp, but has become a substantial part of the national offering in education, destined to affect our thinking and practice in ways not now foreseen.

"Both in the support of general education studies and in its grants to specific institutions, the Corporation works in close accord with the Carnegie Foundation. The Foundation does not itself make grants, but the conclusions reached in its professional studies can and do influence the policies of the sister institution."

The Corporation has published a volume of tributes to Andrew Carnegie commemorating the one hundredth anniversary (1935) of his birth.

Commonwealth Fund, 1935. — "The Fund has appropriated in the year 1934-1935 the sum of $1,574,025.07. Of the total, 58.3 percent has been devoted to the promotion of health. In addition to other work, the Fund has shared the cost of a critical study of public health needs and organization in Massachusetts; an investigation of the educational opportunities presented by internships and residencies in the hospitals of New York; teaching clinics at Tulane; a teaching and research service in tuberculosis which uses the wards of Bellevue Hospital, New York; the established services of the American Public Health Association and the National Organization for Public Health Nursing; the continued work of the National Conference on Nomenclature of Disease; and research in child development at the Neurological Institute, New York.

"The Commonwealth Fund Fellowships for British Students took 16.1 percent of total appropriations; the Fund's mental hygiene activities 13.8 percent. The 11.8 percent which completes the total includes the cost of the Fund's publishing service, legal research, and library grants to the Welfare Council and the family welfare societies of New York, a grant to the Governor's Commission on Unemployment Relief in New York State, and a special underwriting for the expenses of introducing a group payment plan in the Associated Hospital Service of New York."

General Education Board, 1934-35.—Appropriations made during the year amounted to $7,494,150. During 1934-35, the General Education Board carried further those forms of support to American education that were indicated in the plans adopted in April, 1933. Grants were made to various agencies for work in the field of general education, for research in child development, and for improvement of educational opportunity in the southern states. Support of the concentrated programs of the Rockefeller Foundation was also given through the use of funds for the training of personnel. The interest of the Board in opportunities to assist organizations involved in the emergency educational services of the Government was further shown by a series of grants continuing temporarily the support rendered them during the preceding year. Each of these phases of the program now in effect was described in the report for the year 1933-34.

Rockefeller Foundation, 1934.—The Foundation expended $12,679,775 during 1934, distributed in part as follows: public health, $2,200,000; medical sciences, $1,026,200; natural sciences, $1,051,210; social sciences, $1,164,690; emergency grants, $525,500; and the humanities, $749,500.

Research in Scotland.—The annual report (1934-35) of the Scottish Council for Research in Education outlines the work completed and suggests certain projects to be undertaken:

1. The individual testing of a representative sample of the school population by both Binet and performance tests.

2. A scholastic survey of all Scottish children of age 11 plus, or of a sufficiently large representative sample.

3. An inquiry into different types of qualifying examination employed in certain areas and of the substitutes adopted by various Education Committees.

These proposals have met with the approval of the International Institute of Teachers College, Columbia University, and a grant of $10,000 to be paid in four installments, has been promised.

Library Procedure.—The *Library Consultant* is a quarterly issued by the library staff of Teachers College, Columbia University. Copies of any issue for which copies remain will be sent postpaid upon application and receipt of fifteen cents each by the Office of the Library, Teachers College, Columbia University, 525 West 120 Street, New York City. Subscriptions will be taken for $ .50 per year.

The following numbers of the *Library Consultant* have appeared or are scheduled: No. 1, "Book Reviews," April, 1934. No. 2, "Dictionaries," July, 1934. No. 3, "Biography," November, 1934. No. 4, "Government Documents," January, 1935. No. 5, "Library Materials for the Study and Teaching of English," April, 1935. No. 6, "Library Materials for the Study and Teaching of History—Part I," July, 1935. No. 7, "Library Materials for the Study and Teaching of History—Part II," October, 1935. No. 8, "Statistics Needed in Educational Work: References and Notes on Sources," January, 1936. No. 9, "Library Materials for the Study and Teaching of Science,"

April, 1936. No. 10, "Library Materials for the Elementary School Teacher," July, 1936.

Carter Alexander has been active in the preparation of this useful mimeographed publication.

Occupational Index.—A new service, providing a guide to current literature describing occupational opportunities, requirements, and trends, began in January with the appearance of *Occupational Index*, prepared and published by the National Occupational Conference, 551 Fifth Avenue, New York. The conference is a research organization supported by the Carnegie Corporation.

Through correspondence with 500 industrial organizations, professional associations, and research workers, and through examination of 100 magazines and all new books, the Conference will compile each month an annotated and classified bibliography of occupational information useful for purposes of vocational guidance and vocational education. This will be published in periodical form and distributed monthly to subscribers. A special grant from the Carnegie Corporation enables the Conference to offer the service at a subscription rate of five dollars a year.

Arkansas Project.—The Division of Information and Research of the Arkansas State Department is now conducting a Homestead Valuation Survey in the 75 counties of the state as a WPA project. It is the purpose to ascertain the number of homesteads in the state, the assessed valuation of same, and the amount of local and state school taxes that would be lost by exemption of homesteads assessed at $1,000 and $2,500, respectively. Working under Crawford Greene as Director are four non-relief supervisors and 88 relief workers. It will require six months to complete the task. The study was sponsored by Commissioner W. E. Phipps in order to have definite information with which to meet the continuous demands for homestead exemptions.

California Research.—A mimeographed bulletin of the California State Department of Education, prepared by W. E. Morgan, assistant superintendent and chief of the Division of Research and Statistics, covers studies completed, in progress, and proposed in the following fields: adult and continuation education, elementary education, history, junior colleges, libraries, school accounting, finance, housing, secondary education, special education, tenure, teacher training, and textbooks.

Professor A. S. Barr and a number of school men are engaged in a series of intensive studies of teaching aimed to determine if possible the constituents of good teaching and the changes produced in these constituents under supervision. These studies are a part of a long-time program of researches dealing with the teacher as a factor in the teaching-learning situation.

Suggestions for teacher-training classes

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Up-to-date references are annually furnished by the authors.

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